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Surman **J**ournal for **S**cience and **T**echnology
ISSN: Online (2790-5721) - Print (2790-5713)
مجلة صرمان للعلوم والتقنية
Vol **7**, No.1.Jan – Jun. 2025
Pages: 295 ~ 306

Study the Idea of Oblique Incidence on the Surface between two Materials and Derive the Reflection and Transmittance Coefficients to Calculate the Reflection and Transmission of the Field across the Boundary by Polarization

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المخلص:
تتناول هذه الورقة البحثية المبادئ الأساسية لحلول المجال في وسطين شبه لانهائيين خاليين من الفقدان، ومحدودين بحدود مستوية ذات امتداد لانهائي، بالإضافة إلى معاملات الانعكاس والنفاذية، والتي سيتم استخلاصها لتفسير انعكاس المجال وانتقاله عبر الحدود. علاوة على ذلك، يُعد الاستقطاب خاصية من خصائص الموجات المستعرضة، تُحدد الاتجاه الهندسي للتذبذبات في الموجة المستعرضة، يكون اتجاه التذبذب عموديًا على اتجاه حركتها. أما في الموجات الطولية مثل الموجات الصوتية في السوائل أو الغازات فإن إزاحة الجسيمات في التذبذب تكون دائمًا في اتجاه الانتشار.

Abstract:
The basic principles of this paper about the field solutions in two semi-infinite and lossless and lossy media bounded by a planar boundary of infinite extent and, reflection and transmission coefficients will be derived to account for the reflection and transmission of the field by boundary. Moreover, the polarization is a property of transverse waves which specifies the geometrical orientation of the oscillations. In a transverse wave and the direction of the oscillation is perpendicular to the direction of motion of the wave.

Keywords: Non-magnetic media, Amplitude Reflection, Transmission Coefficients at boundary, incident, Reflected, Transmission, Snell's law of reflection.

Submitted: 10/04/2025 *Accepted: 04 /05 /2025*

Introduction
This paper introduces the idea of problem about the oblique incidence at an interface between two materials, and field solutions in two semi-infinite and lossless and lossy media bounded by a planar boundary of infinite extent. Moreover, reflection and transmission coefficients will be derived to account for the reflection and transmission of the field by boundary. These coefficients will be function of the constitutive parameters of the two media, and the direction of wave travel (angle of incidence) and the direction of the electric and magnetic field (wave polarization).

The Equations used to Solve Problems

Non-magnetic media ($\mu_1 = \mu_2 = \mu_0$) we have to write the amplitude reflection and transmission coefficients at boundary $\Gamma_\perp^b$ and $T_\perp^b$ for the case of perpendicular. The reflection $\Gamma_\perp^b$ and transmission $T_\perp^b$ coefficients, and the relation between the incident θ_i and reflected θ_r and transmission (refracted) θ_t and can be obtained by applying the boundary conditions on the continuity of the tangential components of the electric and magnetic field as follows:

$B_1\, x \sin\theta i = B_1\, x\ \sin\theta r = B_2\, x\ \sin\theta t \implies$ (***Snell's law of reflection***)

$\sin\theta i = \sin\theta r \quad \implies \quad \acute{\theta\iota} = \acute{\theta r} \quad \implies$ (***Snell's law of reflection***)

Also, $\sqrt{\mu 1 \varepsilon 1}\ \sin\theta 1 = \sqrt{\mu 2 \varepsilon 2}\ \sin\theta 2 \quad \implies$ (***GSL***)

For Non-magnetic field $\implies$ $n1\sin\theta i = n2\sin\theta t \implies$ (***Snell's law of reflection)***

Using $\quad 1 + \Gamma_\perp^b = T_\perp^b$ --------------- (1)

$$\frac{\cos\theta i}{\eta 1}(-1+\Gamma_\perp^b) = \frac{\cos\theta i}{\eta 2} T_\perp^b \quad \text{-------------- (2)}$$

The first state Equation for Solving system

Where solving equation (1) and (2) simultaneously for $\Gamma_\perp^b$ ***and*** $T_\perp^b$ we can get that:

$$\Gamma_\perp^b = \frac{E_\perp^r}{E_\perp^i} = \frac{\eta 2\ \cos\theta i - \eta_1\ \cos\theta t}{\eta 2\ \cos\theta i + \eta_1 \cos\theta t} = \frac{\sqrt{\frac{\mu 2}{\varepsilon 2}}\cos\theta i - \sqrt{\frac{\mu 1}{\varepsilon 1}}\cos\theta t}{\sqrt{\frac{\mu 2}{\varepsilon 2}}\ \cos\theta i + \sqrt{\frac{\mu 1}{\varepsilon 1}}\ \cos\theta t}$$

$$T_\perp^b = \frac{E_\perp^t}{E_\perp^i} = \frac{2\eta 2\ \cos\theta i}{\eta 2 \cos\theta i + \eta 1 \cos\theta t} = \frac{2\sqrt{\frac{\mu 2}{\varepsilon 2}}\cos\theta i}{\sqrt{\frac{\mu 2}{\varepsilon 2}}\ \cos\theta i + \sqrt{\frac{\mu 1}{\varepsilon 1}}\ \cos\theta t}$$

Hence the $\Gamma_\perp^b$ and $T_\perp^b$ are usually referred to as the plane wave Fresnel reflection and transmission coefficients for perpendicular polarization. Since for most dielectric media (excluding ferromagnetic material) **(** $\mu_1 = \mu_2 = \mu_0$**)** reduce by also utilizing to:

$$\Gamma_\perp^b = \frac{\cos\theta i - \sqrt{\frac{\varepsilon 2}{\varepsilon 1}}\sqrt{1-(\frac{\varepsilon 1}{\varepsilon 2})\sin\theta i^2}}{\cos\theta i + \sqrt{\frac{\varepsilon 2}{\varepsilon 1}}\sqrt{1-(\frac{\varepsilon 1}{\varepsilon 2})\sin\theta i^2}} \quad \underline{\textit{Reflection}} \text{ ------------------- (3a)}$$

$$T_\perp^b = \frac{2\cos\theta i}{\cos\theta i + \sqrt{\frac{\varepsilon 2}{\varepsilon 1}}\sqrt{1-(\frac{\varepsilon 1}{\varepsilon 2})\ \sin\theta i^2}} \quad \underline{\textit{Transmission}} \text{ -------------- (3b)}$$

Where **n1** = $\sqrt{\varepsilon 1}$ and **n2** = $\sqrt{\varepsilon 2}$.

By using invoking Snell's law, we write two equations as fowling

And $\varepsilon 1 = n1^2$ & $\varepsilon 2 = n2^2 \implies \frac{\varepsilon 1}{\varepsilon 2} = \frac{n1^2}{n2^2}$

The reflection equation is $\Gamma_\perp^b = \dfrac{\cos\theta i - \frac{n2}{n1}\sqrt{1-(\frac{n1}{n2}^2)\sin\theta i^2}}{\cos\theta i + \frac{n2}{n1}\sqrt{1-(\frac{n1}{n2}^2)\sin\theta i^2}}$

The transmission equation is $T_\perp^b = \dfrac{2\cos\theta i}{\cos\theta i + \frac{n2}{n1}\sqrt{1-(\frac{n1}{n2}^2)\ \sin\theta i^2}}$

The second state Equation for Solving system by using Matlab

By using Matlab we want to make a plot of $|\Gamma_{\perp}^{b}|$ and $|T_{\perp}^{b}|$ versus the incident angle $\acute{\theta\iota}$ for the choices $n2/n1$ = 2, 4, 6, and 8.Plots of $|\Gamma_{\perp}^{b}|$ and $|T_{\perp}^{b}|$ of equation [3a] and [3b] for $n_1 = \sqrt{\boldsymbol{\varepsilon 1}}$ and $n_2 = \sqrt{\boldsymbol{\varepsilon 2}}$ andn_2/n_1 = 2, 4, 6, and 8, and function of θ_i are shown in figures. It is apparent that as the relative ratio of n_2/n_1increases, the magnitude of the reflection coefficient increases whereas that of the transmission coefficient decreases. This is expected since large ratios of interface. In addition, it is observed that for $n_2 > n_1$ the magnitude of the reflection coefficient never vanishes regardless of the n_2/n_1 ratio or the angle of the reflection. When $n_2/n_1 > 1$ both $\Gamma_{\perp}^{b}\, and\ T_{\perp}^{b}$ are real with $\Gamma_{\perp}^{b}$ being negative and $T_{\perp}^{b}$ being positive for all angles of incidence. Therefore, as a function of θ_i the phase of $\Gamma_{\perp}^{b}$ is equal to $1\dot{8}0$ as shown in figures, and that of the transmission coefficient $T_{\perp}^{b}$ is zero as shown in figures. When n_2/n_1= 1 the reflection coefficient vanishes and the transmission coefficient reduces to unity. When $n_2/n_1 < 1$ then both $\Gamma_{\perp}^{b}$ and $T_{\perp}^{b}$ become complex as shown in figures.

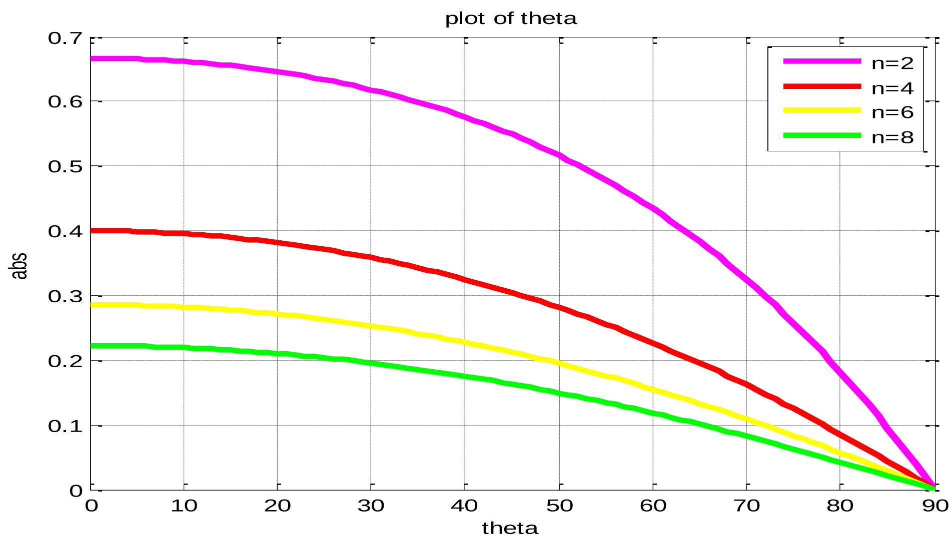

Fig.1. Transmission coefficient

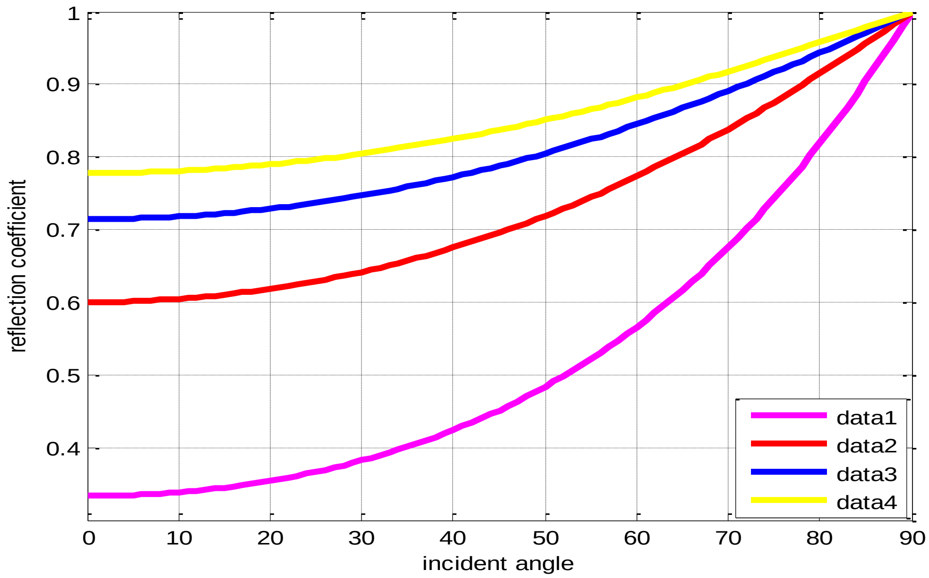

Fig.2. Reflection coefficient

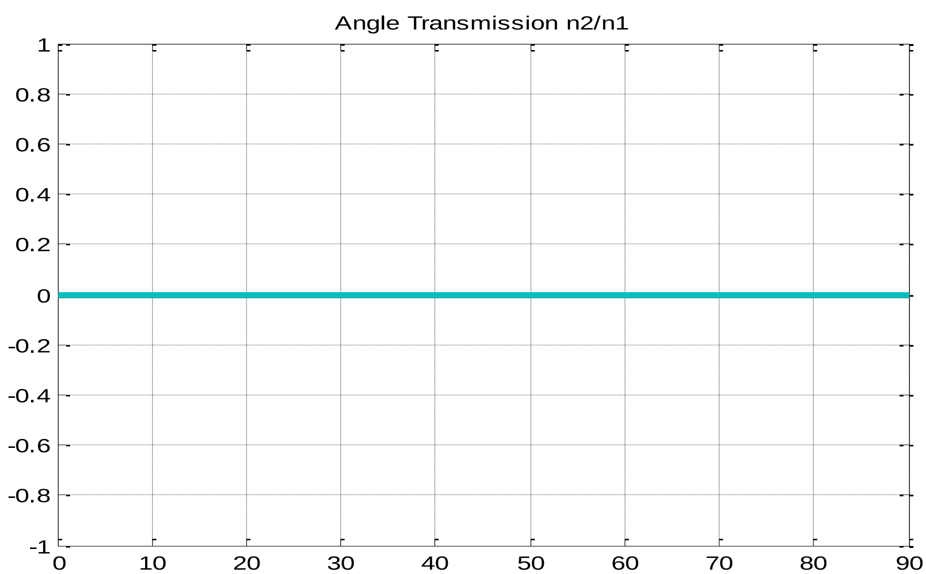

Fig.3. Angle transmission n_2/n_1

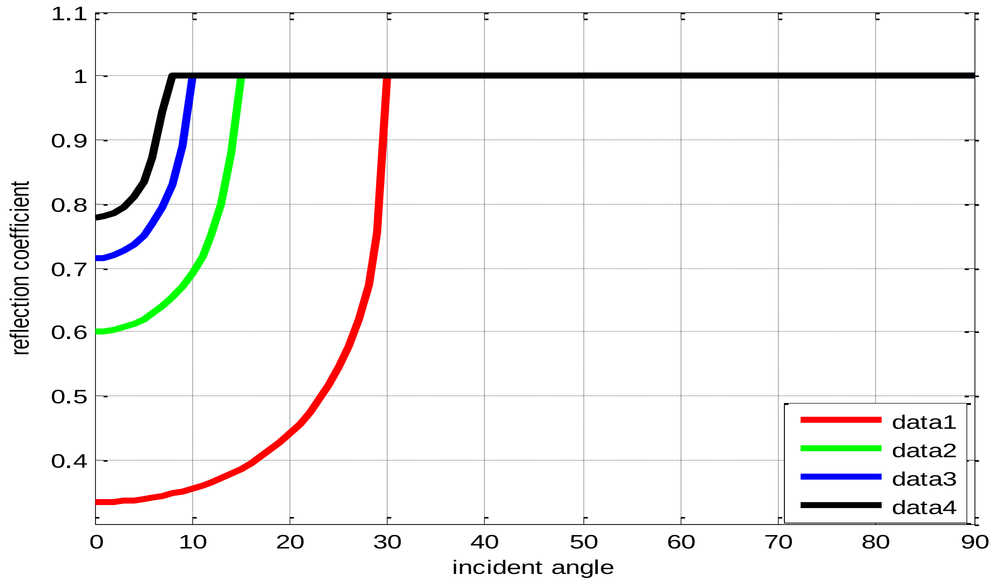

Fig.4. Angle Reflection coefficient n_2/n_1

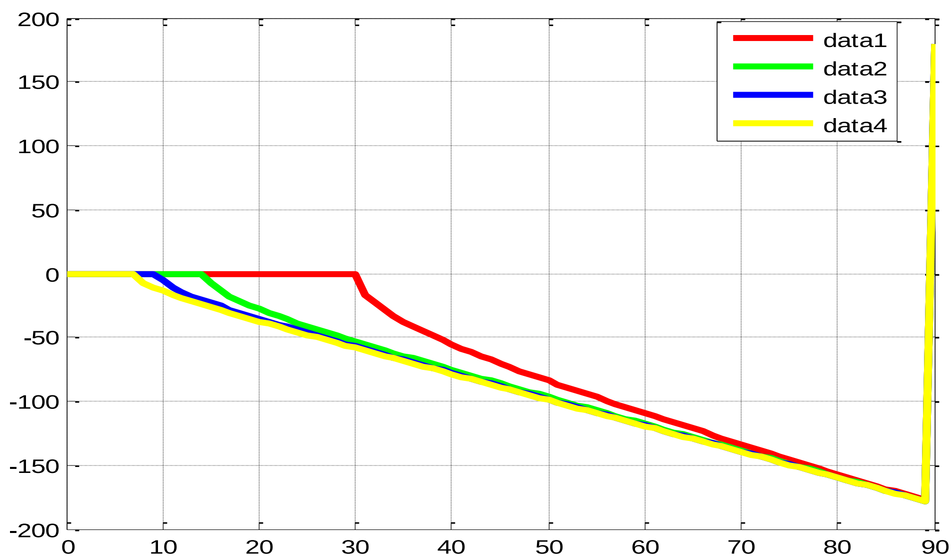

Fig.5. Angle transmission n_2/n_1

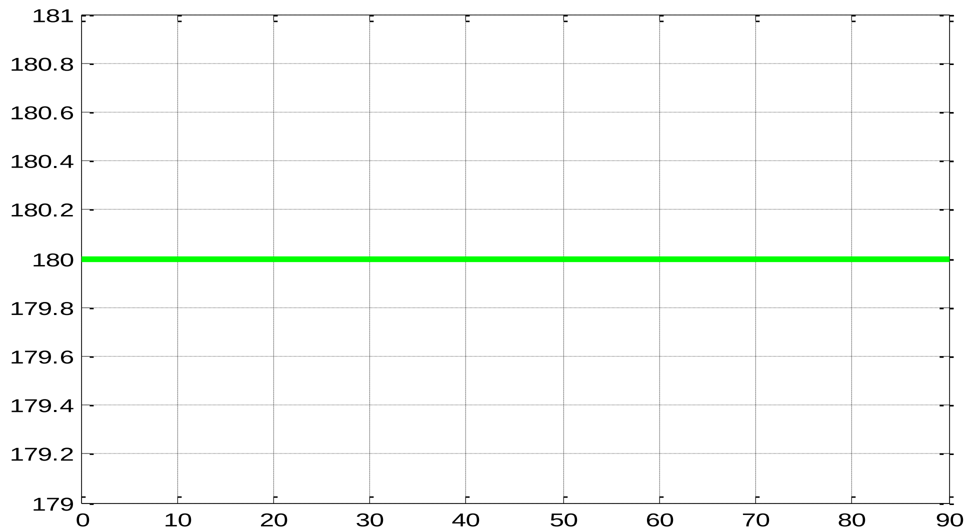

Fig.6. Angle Reflection n_2/n_1

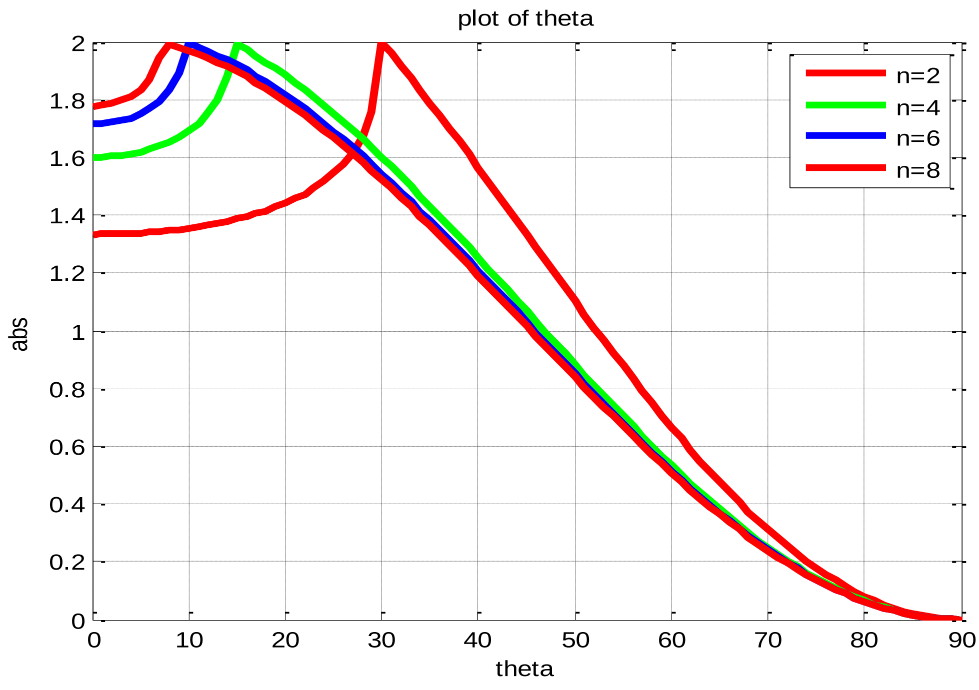

Fig.7. Plot of theta (n=2,n=4,n=6,n=8)

By using Snell's Law

which can be stated as $(nA\ Sin\theta\ A\ =\ nB\ Sin\theta\ B)$ predicts how the ray will change direction as it passes from one medium into another, or as it is reflected from the interface between two Media and the Angles in this equation are referenced to a surface normal, as is illustrated below:

$B_1\, x \sin\theta i = B_1\, x\ \sin\theta r = B_2\, x\ \sin\theta t\ \Longrightarrow$ (*Snell's law of reflection*)

$\sin\theta i = \sin\theta r \quad \Longrightarrow \quad \theta_r = \theta_i \quad \Longrightarrow \quad$ (*Snell's law of reflection*)

$$\boldsymbol{\Gamma}_{\parallel}^{b} = \frac{-\eta 1 \cos\theta i - \eta_1 \cos\theta t}{\eta 1 \cos\theta i + \eta_1 \cos\theta t} = \frac{-\sqrt{\frac{\mu 1}{\varepsilon 1}}\cos\theta i - \sqrt{\frac{\mu 2}{\varepsilon 2}}\cos\theta t}{\sqrt{\frac{\mu 1}{\varepsilon 1}}\cos\theta i + \sqrt{\frac{\mu 2}{\varepsilon 2}}\cos\theta t}$$

$$\mathrm{T}_{\parallel}^{b} = \frac{2\eta 2 \cos\theta i}{\eta 1 \cos\theta i + \eta 2 \cos\theta t} = \frac{2\sqrt{\frac{\mu 2}{\varepsilon 2}}\cos\theta i}{\sqrt{\frac{\mu 1}{\varepsilon 1}}\cos\theta i + \sqrt{\frac{\mu 2}{\varepsilon 2}}\cos\theta t}$$

Hence, the $\boldsymbol{\Gamma}_{\parallel}^{b}$ and $\mathrm{T}_{\parallel}^{b}$ are usually referred to as the plane wave Fresnel reflection and transmission coefficients for parallel polarization. And excluding ferromagnetic material reduce:

$$\boldsymbol{\Gamma}_{\parallel}^{b} = \frac{-\cos\theta i + \sqrt{\frac{\varepsilon 1}{\varepsilon 2}}\sqrt{1-(\frac{\varepsilon 1}{\varepsilon 2})\sin\theta i^2}}{\cos\theta i + \sqrt{\frac{\varepsilon 2}{\varepsilon 1}}\sqrt{1-(\frac{\varepsilon 1}{\varepsilon 2})\sin\theta i^2}}$$

$$\boldsymbol{\Gamma}_{\parallel}^{b} = \frac{-\cos\theta i + \frac{n1}{n2}\sqrt{1-\frac{n1^2}{n2^2}\sin\theta i^2}}{\cos\theta i + \frac{n1}{n2}\sqrt{1-\frac{n1^2}{n2^2}\sin\theta i^2}} \quad \underline{\textit{Reflection}}\text{------------------ (4a)}$$

$$\mathrm{T}_{\parallel}^{b} = \frac{2\sqrt{\frac{\varepsilon 1}{\varepsilon 2}}\cos\theta i}{\cos\theta i + \sqrt{\frac{\varepsilon 1}{\varepsilon 2}}\sqrt{1-(\frac{\varepsilon 1}{\varepsilon 2})\sin\theta i^2}}$$

$$\mathrm{T}_{\parallel}^{b} = \frac{2\frac{n1}{n2}\cos\theta i}{\cos\theta i + \frac{n1}{n2}\sqrt{1-\frac{n1^2}{n2^2}\sin\theta i^2}} \quad \underline{\textit{Transmission}}\text{ ----------- (4b)}$$

Plots of $\boldsymbol{\Gamma}_{\parallel}^{b}$ and $\mathrm{T}_{\parallel}^{b}$ for equations [4a] and [4b] when $n_1 = \sqrt{\boldsymbol{\varepsilon 1}}$ and $n_2 = \sqrt{\boldsymbol{\varepsilon 2}}$ andn_2/n_1 = 2, 4, 6, and a function of θ_i are shown in figures, and the polarization there is an angle where the reflection coefficient does vanish and the angle is referred to as the Brewster and it increases toward $\dot{9}0$ as the ratio of n_2/n_1 becomes larger. For $n_2/n_1 > 1$ reflection and transmission are both real and $\boldsymbol{\Gamma}_{\parallel}^{b}$ is negative, indicating a $1\dot{8}0$ phase as function of incidence angle and $\mathrm{T}_{\parallel}^{b}$ is positive representing a $\dot{0}$ phase as shown in figures.

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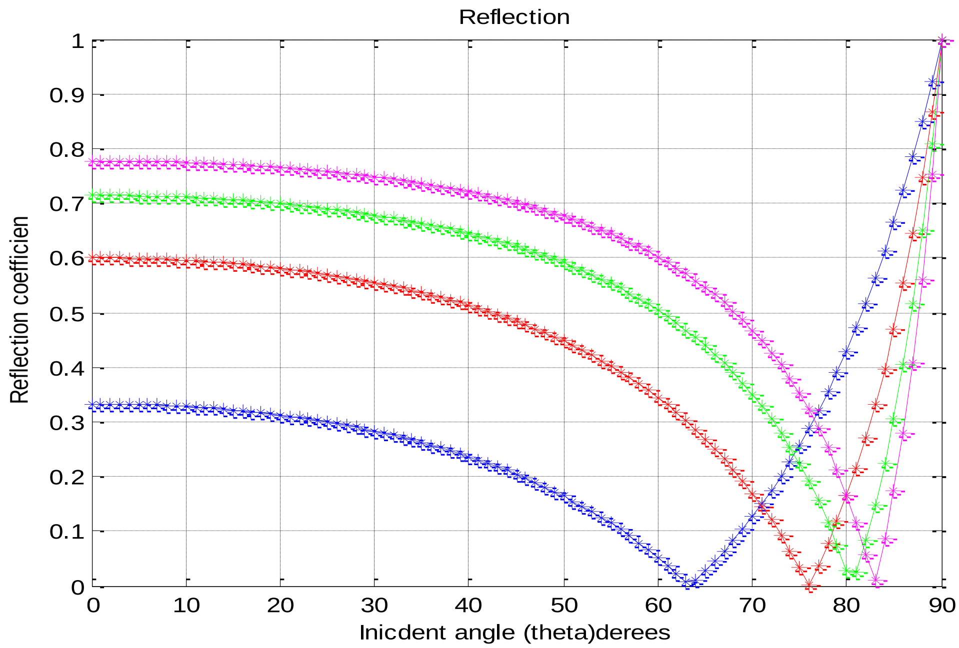

Fig.8. Reflection coefficient $\boldsymbol{\Gamma}_{\parallel}^{b}$

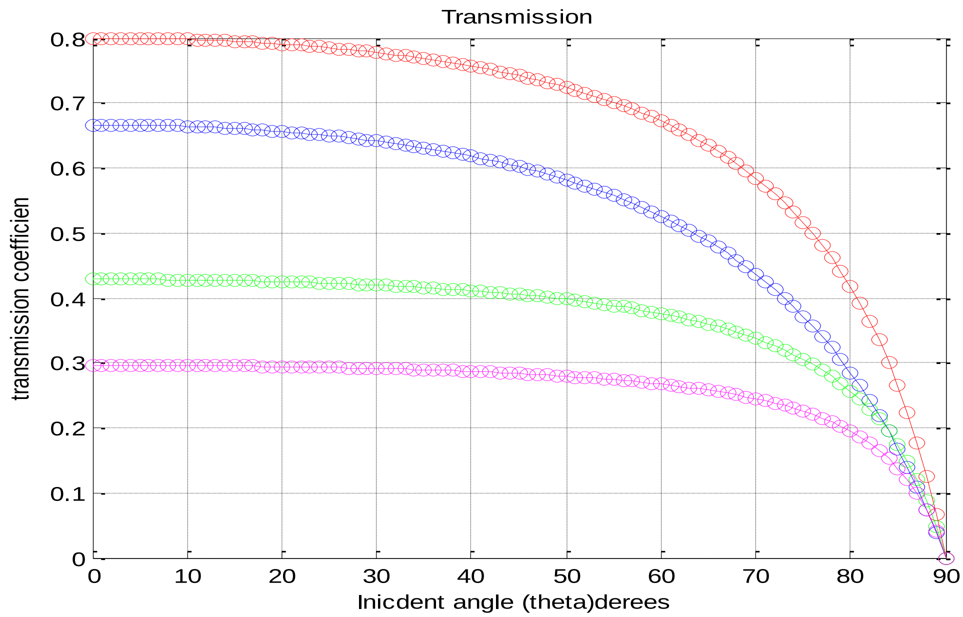

Fig.9. Transmission coefficient $\mathrm{T}_{\parallel}^{b}$

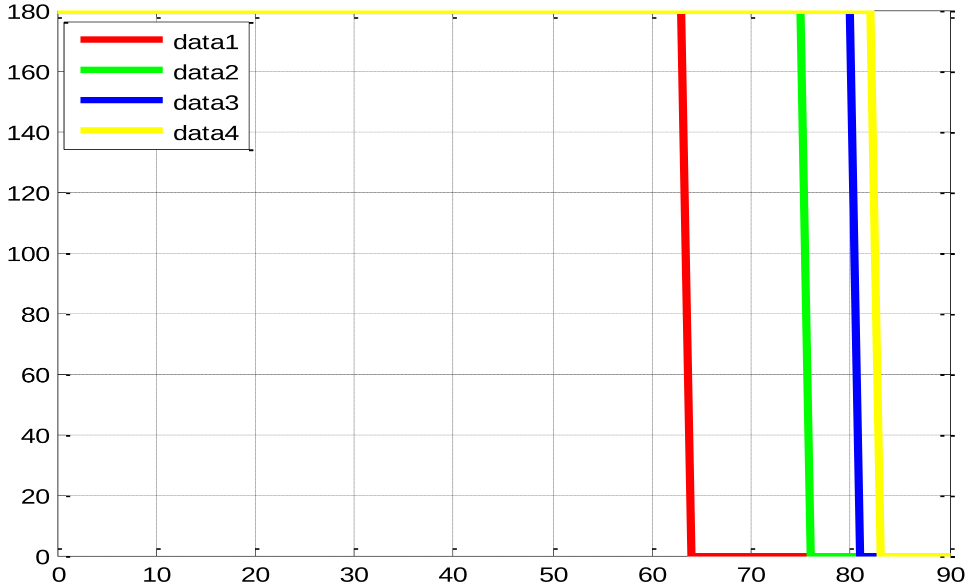

Fig.10. Angle transmission n_2/n_1

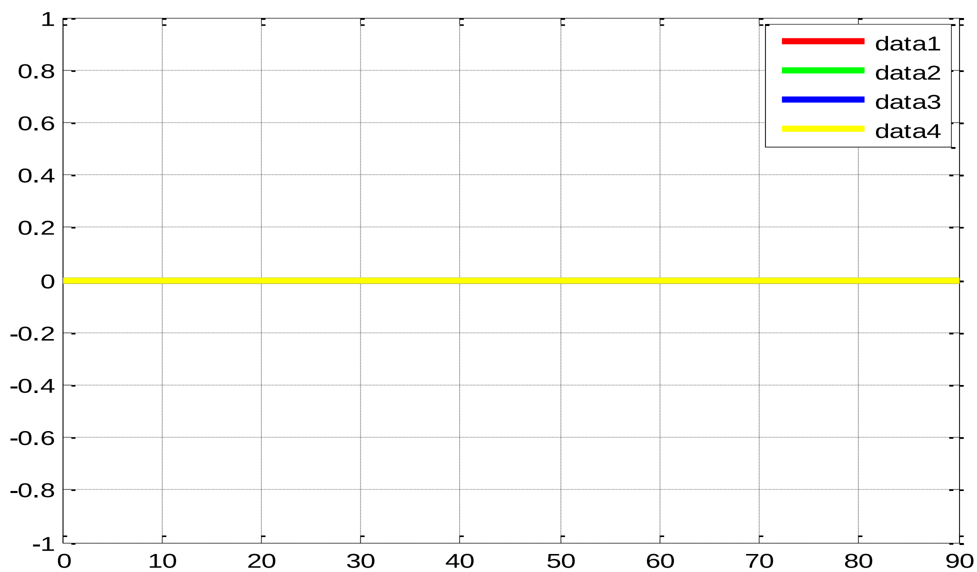

Fig.11. Angle transmission n_2/n_1

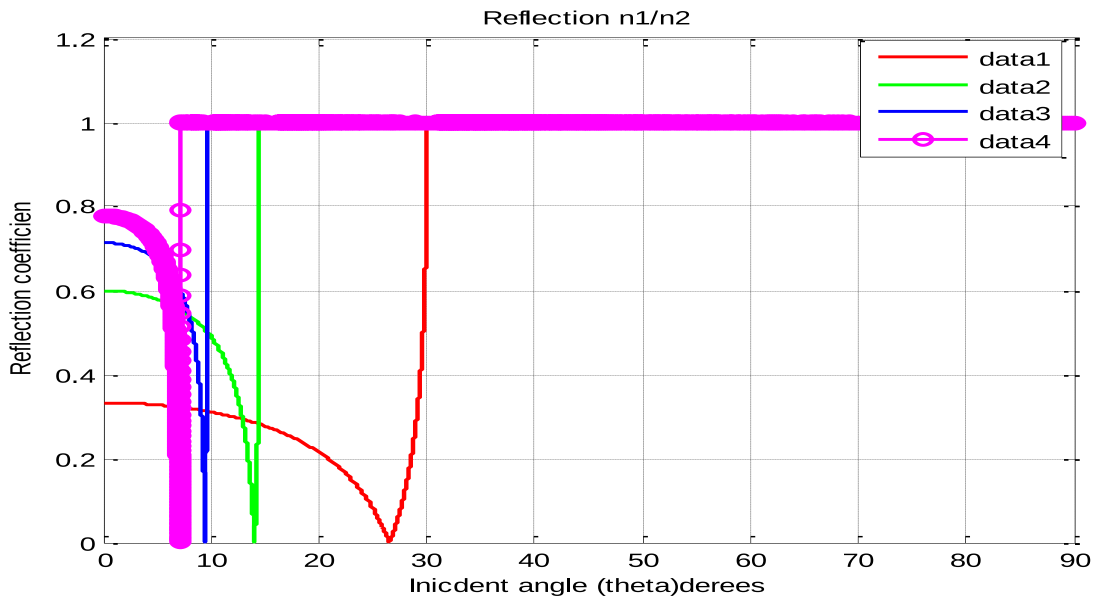

Fig.12. Reflection coefficient $\boldsymbol{\Gamma_{\parallel}^{b}}$

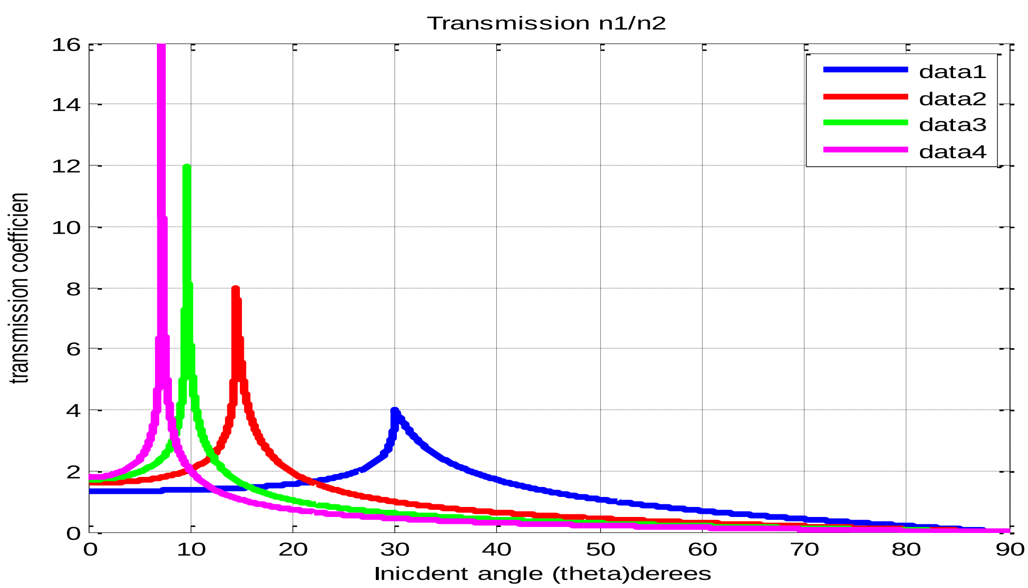

Fig.13. Transmission coefficient $\mathrm{T}_{\parallel}^{\boldsymbol{b}}$

Here, we will find the mathematically check for $\boldsymbol{\Gamma_{\perp}^{b}}$ **,** $\mathrm{T}_{\perp}^{\boldsymbol{b}}$ **,** $\mathrm{T}_{\parallel}^{\boldsymbol{b}}$ **,** $\boldsymbol{\Gamma_{\parallel}^{b}}$

Since, the $\boldsymbol{\cos\theta t} = \sqrt{\boldsymbol{1-\sin\theta t^{2}}} = \sqrt{\boldsymbol{1-\frac{\mu1\varepsilon1}{\mu2\varepsilon2}\sin\theta j^{2}}} = -\mathbf{j}\sqrt{\boldsymbol{\frac{\mu1\varepsilon1}{\mu2\varepsilon2}\sin\theta j^{2}-1}}$

Now, we will check for $\boldsymbol{\Gamma_{\perp}^{b}}$ and $\mathrm{T}_{\perp}^{\boldsymbol{b}}$

$$\Gamma_{\perp}^{b} = \frac{\frac{n1}{n2}\cos\theta i + j\sqrt{\frac{\mu1\varepsilon1}{\mu2\varepsilon2}\sin 2\theta j - 1}}{\frac{n1}{n2}\cos\theta i - j\sqrt{\frac{\mu1\varepsilon1}{\mu2\varepsilon2}\sin 2\theta j - 1}} \qquad \textit{where} \quad \mu1 = \mu2 = \mu0$$

$$\Gamma_{\perp}^{b} = \frac{\frac{n1}{n2}\cos\theta i + j\sqrt{\frac{n1^2}{n2^2}\sin 2\theta j - 1}}{\frac{n1}{n2}\cos\theta i - j\sqrt{\frac{n1^2}{n2^2}\sin 2\theta j - 1}} \text{ --------------- (5)}$$

$$\mathrm{T}_{\perp}^{b} = \frac{2\frac{n1}{n2}\cos\theta i}{\frac{n1}{n2}\cos\theta i - j\sqrt{\frac{\mu1\varepsilon1}{\mu2\varepsilon2}\sin 2\theta j - 1}} \text{ ---------------- (6)}$$

$$\Gamma_{\parallel}^{b} = \frac{-\cos\theta i - j\frac{n1}{n2}\sqrt{\frac{n1^2}{n2^2}\sin 2\theta j - 1}}{\cos\theta i - j\frac{n1}{n2}\sqrt{\frac{n1^2}{n2^2}\sin 2\theta j - 1}} \text{ ------------------- (7)}$$

$$\mathrm{T}_{\parallel}^{b} = \frac{2\frac{n1}{n2}\cos\theta j}{\cos\theta i - j\frac{n1}{n2}\sqrt{\frac{n1^2}{n2^2}\sin 2\theta j - 1}} \text{ ------------------ (8)}$$

We have to check mathematically to see what happens to results $\Gamma_{\perp}^{b}$, $\mathrm{T}_{\perp}^{b}$, $\mathrm{T}_{\parallel}^{b}$, $\Gamma_{\parallel}^{b}$ when the incident angle θi exceeds the critical angle between the two media. For to find each $\Gamma_{\perp}^{b}$, $\mathrm{T}_{\perp}^{b}$, $\mathrm{T}_{\parallel}^{b}$, $\Gamma_{\parallel}^{b}$ as complex numbers.

Conclusion

According to the results we got from this papers by using Matlab to make a plot of $|\Gamma_{\perp}^{b}|$ and $|\mathrm{T}_{\perp}^{b}|$ where for $n_1 = \sqrt{\varepsilon 1}$ and $n_2 = \sqrt{\varepsilon 2}$ andn_2/n_1 = 2, 4, 6, and 8 and $\frac{\varepsilon 1}{\varepsilon 2} = \frac{n1^2}{n2^2}$ and we have to represent rarer to denser with four plots as we shown in figures, also with plot phase angles θ_{Γ} and θ_{T} vesus θ_i. As a result of which a function of θ_i of the reflection coefficient is equal to $1\dot{8}0$ and that of the transmission coefficient is zero. Also, the phase angles θ_{Γ} and θ_{T} vesus θ_i become complex number or output for each n_1/n_2 and phase angles n_2/n_1 for reflection and transmission coefficient are $\dot{0}$ and$1\dot{8}0$. In addition, we have to calculated for the case of parallel $\Gamma_{\parallel}^{b}$ and $\mathrm{T}_{\parallel}^{b}$ and generated two graphs with four plots also as we shown in figures, and this will be represented rarer to denser propagation. The results for n_1/n_2 that will be represent denser to rarer propagation.

Note:

Mathematically, we can see that every reflection coefficient, transmission coefficient and the case of parallel will be complex number.

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