sjst.scst.edu.ly
**S**urman **J**ournal for **S**cience and **T**echnology
ISSN: Online (2790-5721) - Print (2790-5713)
مجلة صرمان للعلوم والتقنية
Vol **6**, No.**2**, Jun - Dec. 2024
Pages: 121 - 138

# Improve Feedback Linearization Control For SISO Nonlinear Systems

**Abdunaser M. Abdusamad[1*], Farag I. k. Mousa[1]**

[1]Department of Electrical Engineering, Faculty of Engineering, Azzaytuna University, Libya.

*Corresponding author, e-mail: n.abed@azu.edu.ly

## Abstract

*Feedback linearization is a powerful technique used in control systems to transform the dynamics of nonlinear systems into a linear form, making them easier to analyze and control. However, dealing with highly nonlinear systems can be challenging and complicated. This paper aims to address this issue by proposing an improved approach to the feedback linearization method. To enhance the feedback linearization control of single-input single output (SISO) nonlinear systems, the paper explores two main strategies. The first approach involves adjusting the control gains in conjunction with other parameters to optimize the control performance. This allows for fine-tuning the system's behavior and response to achieve desired objectives. The second approach focuses on evaluating the performance of the feedback linearization control through simulations under diverse scenarios, disturbances, and reference inputs. By conducting these simulations, the researchers can thoroughly analyze how the system behaves and performs under various conditions. Importantly, throughout these adjustments and simulations, ensuring system stability remains a crucial consideration. The paper delves into two specific techniques for designing feedback linearization control: input-output linearization and input-state linearization. Both techniques offer distinct advantages and trade-offs depending on the system requirements and characteristics. By employing these techniques, the designer aims to achieve the desired behavior and performance of the SISO nonlinear system.*

## خلاصة البحث

تعتبر التقنية الخطية الراجعة؛ من أفضل التقنيات المستخدمة في أنظمة التحكم الحديث؛ لتحويل ديناميكية الانظمة الغير خطية الى أنظمة خطية. وذلك بسبب أن التعامل مع الانظمة الغير خطية يكون صعبا ومعقد للغاية، بينما من السهل التعامل مع الأنظمة الخطية وتحليل بياناتها والتحكم بها. ونركز في هذه الورقة على دراسة هذه المشكلة ومعالجتها من خلال إتباع نهج 'محسن لطريقة التقنية الخطية الراجعة، التي من شأنها أن تعزز التحكم الخطي الأنظمة الغير خطية ذات المدخلات والمخرجات الفردية. وتهدف هذه الورقة لتحسين سلوك الأنظمة الغير خطية وزيادة استجابتها لأوامر التحكم وذلك بأتباع نهجين مختلفين هما على النحو التالي: يقترح النهج الاول تعديل تغذية هذه الأنظمة بالتزامن مع بعض المعلمات الاخرى، في حين 'يركز النهج الثاني على تحسين سلوك النظام من خلال عمليات المحاكاة في ظل سيناريوهات ومدخلات مرجعية متنوعة. ومن خلال أجراء عمليات هذه المحاكاة، يمكن للباحثين أجراء تحليل شامل لكيفية تعامل النظام وأدائه في ظل ظروف مختلفة. في حين يظل استقرار النظام أحد الاعتبارات الحاسمة مهما كانت طريقة التعديل المقترحة. وثم التركيز في هذه الورقة على استخدام طريقتين مختلفتين وهما طريقة التصميم الخطي الراجع باستخدام المدخلات والمخرجات، وطريقة التصميم الخطي الراجع باستخدام المدخلات وحالة النظام المستخدمة. وحققت كلتا الطريقتين نجاحات ملموسة ساهمت بشكل جيد في تحسين أداء الانظمة اللاخطية التي أجريت عليها الدراسة في هذه البحث.

***Keywords**: Feedback Linearization Control, Tracking Control, Nonlinear SISO Systems, System Stability, System Disturbances, System Optimization.*

## 1. Introduction

In the feedback linearization method, the focus is to stabilize and control nonlinear systems through the transformation of their dynamics into a linear form. According to [1], this method gives us the opportunity to use linear control techniques on nonlinear systems. This approach aims to eliminate the nonlinearity in the system dynamics by finding a suitable change of variables [1] and [2].

According to [3], in continuous - time state space models, the nonlinear system can be represented in the following form,

$$\left.\begin{aligned} \dot{x} &= f(x) + g(x)u, \\ y &= h(x). \end{aligned}\right\} \quad (1)$$

where: x is an n-dimensional state variables vector; u is an m-dimensional control vector of manipulated input variables; y is an m-dimensional output variable vector; f(x) is an n-dimensional vector of nonlinear function; g(x) is an $(n \times m)$-dimensional matrix of nonlinear functions; and h(x) is an m-dimensional vector of nonlinear functions. For SISO case $m = 1$. The advantage of the feedback linearization method is its ability to produce a linear model that accurately represents the original nonlinear model over a wide range of operating conditions. The process is divided into two operation steps. In the first step, the system's nonlinear coordinates are modified; and in the second step, nonlinear state feedback is implemented [4] and [5]. This work focuses on local feedback linearization, which means that the coordinate transformation and control law can only be defined locally. This is because we need to eliminate complications associated with the global problem [6]. The feedback linearization method is generally based on two main approaches: input-output linearization and input-state linearization. The input-output approach is intended to define a linear path between transformed inputs (v) and actual outputs (y). The next step is designing a linear controller for the linearized input-output model. However, in most cases of this method, there is a subsystem that cannot be linearized [7]. While in the second approach which is input state linearization, the purpose is to linearize the map between the transformed inputs and the entire vector of transformed state variables [1]-[3]. This goal can be obtained by creating artificial outputs (w) that generate a feedback linear model with state dimension $r = n$. The design controller formed with this approach is complex because the map between transformed inputs and original outputs (y) is generally nonlinear. As a result of this weakness, the input-state linearizable method less used compared with the input-output linearizable method [5].

After the feedback linearization process, the system model becomes linear in the form:

$$\left.\begin{aligned} \zeta &= A\dot{\zeta} + Bv, \\ W &= C\zeta \end{aligned}\right\} \quad (2)$$

Where: $\zeta$ is r- dimensional vector of transformed state variables; v and w are m- dimensional vectors of transformed input and output variables respectively; and the matrices A; B and C are simple structures.

Different techniques were applied in this area and yielded diverse results. For example, In [1], the authors explain in detail the principle of feedback linearizing control. In [2], Horacio J. and Marquez explain the analysis and design for nonlinear control systems. Hassan Khalil discusses the principle of adaptive output feedback control of nonlinear systems in [3] and [4]. Feedback linearization families of nonlinear systems are presented by Wang, Jianliang, and W. Rugh in [5]. In [6], Sastry and Shankar presented an analysis, stability, and control of nonlinear systems. While, nonlinear control of Multi-Input-Multi Output (MIMO) system using feedback linearization control method and PD controller for tracking purpose is introduced by Ghozlane, Wafa, and Jilani Knani in [7]. More details about feedback linearization of nonlinear MIMO variables can be found in [8], [9] and [10]. On the other hand, information about the adaptive MIMO nonlinear systems using fuzzy logic control and extreme learning machine can be found in [12] and [13]. In [15] and [16] the authors presented the output feedback linearization of neural network-based ANARX models and nonlinear control for output voltage regulation of a boost converter with a constant power load respectively.

The remaining sections of the paper are structured as follows: Section II provides an in-depth explanation of two distinct techniques of feedback linearization. These techniques are presented to offer a comprehensive understanding of the principles behind feedback linearization and highlight their significance in addressing nonlinear systems effectively. In Section III, the problem statement is outlined, focusing on two specific single-input single-output (SISO) nonlinear systems. The section delves into the details of these systems, discussing their characteristics and complexities. Section IV describes the simulation process applied using the principal input-output and input-state feedback linearization techniques. The section explains the methodology employed and the parameters considered during the simulation. Furthermore, the obtained results, including the system's responses, are displayed and analyzed in detail. This section serves as a critical evaluation of the effectiveness and suitability of the feedback linearization techniques in addressing the identified problems. Finally, Section V provides the concluding remarks of this work. It summarizes the main findings, discusses the implications of the results, and highlights the contributions made by the study. The section also offers insights into potential future research directions and areas where further improvements can be made. Overall, this section serves as a comprehensive wrap-up of the paper, emphasizing the significance and implications of the presented work.

## 2. The Principle Of Feedback Linearization Method

In this section, we present two techniques that can work in this area which are the input-output feedback linearization technique and the input-state linearization technique.

### 2.1. Input-Output Feedback Linearization Technique

The discussion in this subsection focuses on the concept of linearization of input-output feedback in nonlinear systems. The primary aim of feedback linearization is to establish a linear relationship between the output Y and the new input V as shown Figure 1. For SISO systems in Equation 1, f; g and h are sufficiently smooth in a domain $U < R^n$. The mappings $f: U \rightarrow R^n$ and $g: U \rightarrow R^n$ are called vector fields on U [11] and [14]. Referring to Equation 1, and by computing the first derivative of the output y with respect to x.

$$\dot{y} = \frac{\partial h}{\partial x}\dot{x} = \frac{\partial h}{\partial x}[f(x) + g(x)u] \triangleq D_f h(x) + D_g h(x)u, ------ \quad (3)$$

where:

$$\left.\begin{aligned} D_f h(x) &= \frac{\partial h}{\partial x}[f(x)], \\ D_g h(x) &= \frac{\partial h}{\partial x}[g(x)] \\ D_g D_f h(x) &= \frac{\partial D_f h}{\partial x} g(x) \end{aligned}\right\} -------------------- \quad (4)$$

Then:

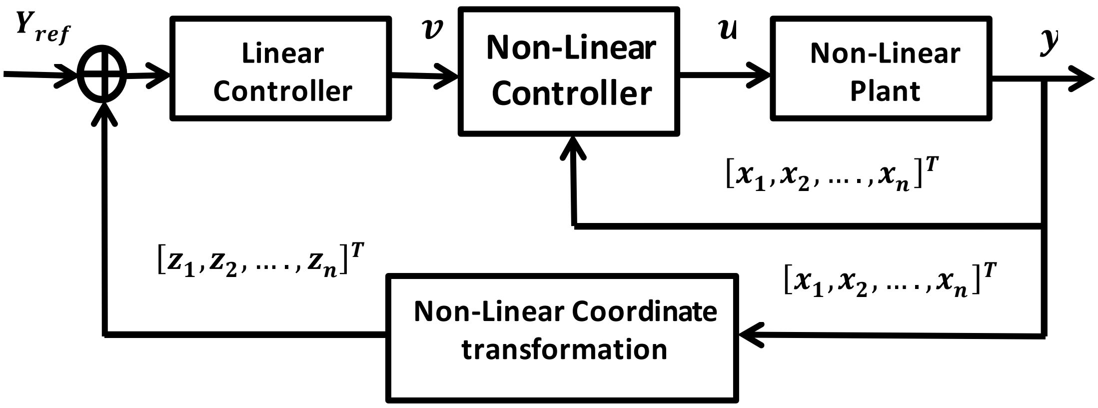



*Figure. 1: Input-output feedback linearization technique [7]*

$$\left.\begin{aligned} D_f^0 h(x) &= h(x),\\ D_f^2 h(x) &= D_f D_f h(x) = \frac{\partial(D_f h)}{\partial x} f(x),\\ &\ \vdots\\ D_f^k h(x) &= D_f D_f^{k-1} h(x) = \frac{\partial\left(D_f^{k-1} h\right)}{\partial x} f(x). \end{aligned}\right\} \text{----------} \quad (5)$$

**Assumption 1**: In Equation 5 if $D_g h(x) u = 0$, then $\dot{y} = D_f h(x)$ ( independent of u).

In order to expand the concept, we have a necessity to compute the second derivative and higher.

$$\left.\begin{aligned} y^{(2)} &= \frac{\partial D_f h}{\partial x}[f(x) + g(x)u]\\ &= D_f^2 h(x) + D_g D_f h(x) u, \end{aligned}\right\} \text{---------------} \quad (6)$$

again from the assumption 1, if $D_g D_f h(x) u = 0$, then $y^2 = D_f^2 h(x)$ (again independent of u). Repeating the calculations for higher derivative.

**Assumption 2**: , if $D_g D_f^{i-1} h(x) = 0, i = 1, 2, \dots, r-1, D_g D_f^{r-1} \neq 0$, then u does not appear in $y, \dot{y}, \dots\dots\dots, y^{r-1}$.

the equation 6 is modified as:

$$y^{(r)} = D_f^r h(x) + D_g D_f^{r-1} h(x) u \text{---------------} \quad (7)$$

finally,

$$u = \frac{1}{D_g D_f^{r-1} h(x)}\left[-D_f^r h(x) + v\right]. \text{---------------} \quad (8)$$

By substituting with the value of u in the nonlinear system that represented in Equation 8, the nonlinear system becomes input-output linearizable and reduces to $y^r = v$ i.e. chain of r integrator.

### *Stability Analysis:*

Stability analysis is an essential aspect of the input-output feedback linearization technique. As we know, stability is not the primary objective of input-output feedback linearization, but the stability of the closed-loop system must be ensured when the linearized control design is applied [6]. According to [1],[2], [3] and [6].

**Lemma 1:** The nonlinear system defined in Equation 1, is said to be relative degree $m, 1 \leq m \leq n$ in region $U_0 < U$ if

$$D_g D^{i-1} h(x) = 0, \quad i = 1,2, \ldots\ldots, m-1.$$
$$D_g D^{m-1} h(x) \neq 0 \quad \text{for all } x \in U_0 .$$

**Lemma 2:** The nonlinear system defined in Equation 1, which has relative degree $m \leq n$ in the region U. If $m = n$ then for every $x_0 \in U$, a neighborhood N of $x_0$ exists such that:

$$T(x) = \begin{bmatrix} h(x) \\ D_f h(x) \\ \vdots \\ D_f^{n-1} h(x) \end{bmatrix}, \text{-------------------- (9)}$$

bounded to N, is a diffeomorphism on N [5] and [6].
In addition, if $m < n$, then for each $x_0 \in U$, a neighborhood N of $x_0$ and smooth functions. $\psi_1(x), \ldots\ldots, \psi_{n-m}(x)$ exist such that $\frac{\partial \psi_i}{\partial x} g(x) = 0$, for $1 \leq i \leq (n-m)$ for all $x \in N$, and the matrix.

$$T(x) = \begin{bmatrix} \psi_1(x) \\ \vdots \\ \psi_{n-m}(x) \\ \cdots\cdots \\ h(x) \\ D_f h(x) \\ \vdots \\ D_f^{m-1} h(x) \end{bmatrix} = \begin{bmatrix} \psi(x) \\ \cdots\cdots \\ \varphi(x) \end{bmatrix} = \begin{bmatrix} \zeta \\ \cdots\cdots \\ \xi \end{bmatrix} \text{------- (10)}$$

which is bounded to N is a diffeomorphism on N.
The next step is by taking the derivative for both $\zeta$ and $\xi$ variables, we obtain:

$$\left.\begin{aligned} \dot{\zeta} &= f_0(\zeta, \xi) \\ \dot{\xi} &= A_c \xi + B_c \gamma(x)[u - \alpha(x)] \\ y &= C_c \xi \end{aligned}\right\} \text{------------- (11)}$$

where $\xi \in R_m$, $\zeta \in R^{n-m}$, and $(A_C, B_C, C_C)$ is in controller canonical form representation of a number of $m$ integrators.

$$f_0(\zeta, \xi) = \frac{\partial \psi}{\partial x} f(x)|_{x=T^{-1}}(z), \text{---------------- (12)}$$

where:

$$\gamma(x) = D_g D_f^{m-1} h(x), \text{ and } \alpha(x) = \frac{D_f^m h(x)}{D_g D_f^{m-1} h(x)}.$$

In the normal form, the system is divided into two parts, the external part ξ and the internal part ζ, while the state feedback control is responsible for linearizing the external part.

$$u = \alpha(x) + \beta(x)v. ----------------- (13)$$

The next step is by making $\xi = 0$ Equation 11 becomes:

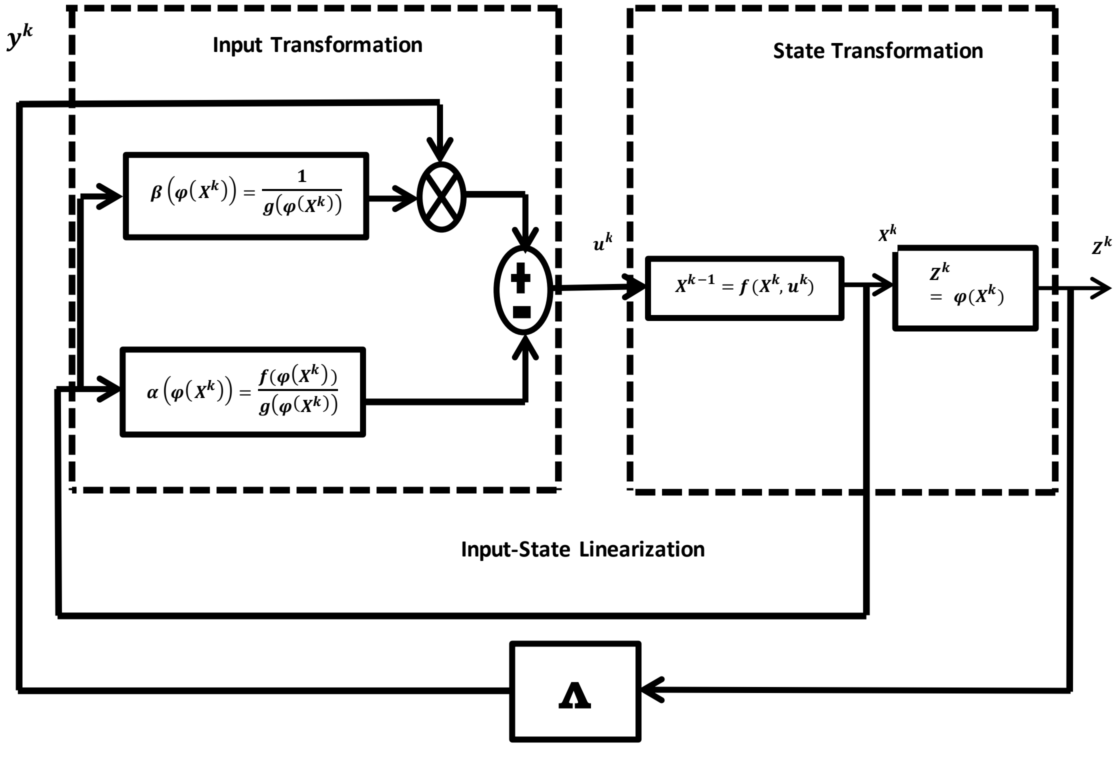
$$\dot{\zeta} = f_0(\zeta, 0) ---------------------- (14)$$

which is called the zero dynamics. Therefore, the system described by Equation 14 which is in minimum phase is asymptotically stable [6] and [7].

## 2.2. Input-State Linearization

In this section, we will briefly discuss the input-state linearization method for nonlinear systems, which has one input (without output) and represented by the following state equation:

*Figure. 2: Input-state linearization technique [17].*




$$\dot{x} = f(x) + g(x)u, ------------------- (15)$$

The system in Equation 15 is input-state linearizable if there is a region ω in $R^n$ that meets the following conditions:

1) The vector fields $\{g,\ ad_{fg}, \ldots\ldots, ad_f^{n-1}g\}$ are linearly independents in the region $\omega$.
2) The set of vectors $\{g,\ ad_{fg}, \ldots\ldots, ad_f^{n-1}g\}$ are involutive. Where $ad_{fg}(x)$ is the **Lie bracket** of $g(x)$ with respect to $f(x)$ and mathematically, the Lie bracket $[g, f]$ can be defined as: $[g; f] = g(x)\nabla f(x) - f(x)\nabla g(x)$, where $\nabla$ represents the gradient

operator, and $g(x)$ and $f(x)$ are vector fields defined on a common domain. From *condition 1*, the vector fields $\{g,\ ad_{fg}, \ldots\ldots, ad_f^{n-1}g\}$ are equivalent to the controllability matrix for linear system $[B \quad AB \quad A^2B \quad \ldots. \quad A^{N-1}B]$, and the involutivity condition indicates that a new vector of linear state through the states feedback can be found [17]. Finally, if both the *conditions 1 and 2* are satisfied then the first state $T_1$ can be found by solving the set of equations: $\nabla T_1 ad_f^i g = 0, i = 1, 2, \ldots\ldots, n-2$, $\nabla T_1 ad_f^{n-1} g \neq 0$ and the state transformation matrix $Z$ can be computed as:

$$Z = T(x) = \begin{bmatrix} T_1 & D_f T_1 & \ldots\ldots. & D_f^{n-1} T_1 \end{bmatrix}^T, ------- \quad (16)$$

and the input transformation:

$$u = \alpha(x) + \beta(x)v. ------------------(17)$$

where

$$\alpha(x) = \frac{D_f^n T_1}{D_g D_f^{n-1} T_1} \quad \text{and} \quad \beta(x) = \frac{1}{D_g D_f^{n-1} T_1}.$$

## 3. Problem Statement

In this section, we present the problem statement by considering a typical example of two SISO nonlinear systems and compare the results obtained. First of all, let us modify Equation 1 in the state form as: Consider the SISO nonlinear system represented by the following state model.

$$\left.\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(x) + g(x)u, \\ y &= x_1 \end{aligned}\right\} ---------------- \quad (18)$$

where $x_1$ and $x_2$ are the system states, u is the control signal and f(x), g(x) are nonlinear functions.

**Example 1:**

Consider the following nonlinear system:

$$m\ddot{z} + n\dot{z} + mn\cos(z) = u --------------- (19)$$

where $m$ and $n$ are constants with values $0.5, 0.05$ respectively, and the objective is to ensure that the output tracks the reference input smoothly with $POT < 11\%$ and $t_{qd} < 0.4$. In order to solve this problem, the first step we will define the following states: $x_1 = z, x_2 = \dot{z}$ and $y = x_1$ then Equation 18 becomes:

$$\left.\begin{aligned} \dot{x}_1 &= \dot{z} = x_2 \\ \dot{x}_2 &= \ddot{z} = -\frac{n}{m}x_2 - n\cos(x_1) + \frac{1}{m}u \\ y &= x_1 \end{aligned}\right\} ----------- (20)$$

The second step is to calculate the time derivative of the output tracking error, $e$ where:

$$e = y_p - y ------------------------ (21)$$

from the equation 20, substitute the value of $y$ with $x_1$.

$$\dot{e} = \dot{y}_p - \dot{x}_1, \text{------------------} (22)$$

from Equation 20, replace $x_1$ and $x_2$, and then take the second derivative of Equation (22).

$$\ddot{e} = \ddot{y}_p - \dot{x}_2, \text{----------------} \quad (23)$$

thence,

$$(24)\ddot{e} = \ddot{y}_p + \frac{n}{m}x_2 + n\cos(x_1) - \frac{1}{m}u. \text{----------}$$

To linearize Equation 24, the controller $u$ can be defined as: $u = nx_2 + nm\cos(x_1) - v$, then by substituting this term into Equation 24.

$$\ddot{e} = \ddot{y}_p - v, \text{-------------------} (25)$$

where $v = \ddot{y}_p + k_1\dot{e} + k_2 e$, and the values of both $k_1$ , $k_2$ are constants. Then, Equation 25 can be rewritten in the form:

$$\ddot{e} = -k_1\dot{e} - k_2 e. \text{----------------} (26)$$

In order to calculate the constant $k$ values, the final step is by comparing Equation 26 with the characteristic equation of the second order system [17] which is:

$$S^2 + 2\zeta\omega_n S + \omega_n^2 = 0. \text{--------------} (27)$$

The values of $k_1$ and $k_2$ in this case are 22 and 256, which are determined using the values of $POT$ and $t_{qd}$.

**Example 2:**

Consider the following nonlinear system:

$$m\ddot{z} + n\dot{z} + mn\ \cos(z^2) = u. \text{-------------} (28)$$

using the same variables as in Example 1, and performing the same steps, we obtain: $k_1 = 22$ and $k_2 = 256$.

## 4. Simulation and Results

The objective of this section is to utilize the two different techniques, input-output feedback linearization and input-state linearization, to solve the problems presented in examples 1 and 2. The objective is to achieve improved control and performance for the systems under consideration. Additionally, the results obtained from both techniques will be compared to assess their effectiveness and suitability for each specific problem. This comparative analysis will provide valuable insights into the strengths and limitations of each approach, aiding in determining the most appropriate technique for addressing similar problems in the future.

**Task (1)**

Simulation using the input-output feedback linearization technique. The simulation results obtained for the problem described in example 1 are shown in Figures 3, 4, while the results obtained for the problem described in example 2 are shown Figures 5 and 6.

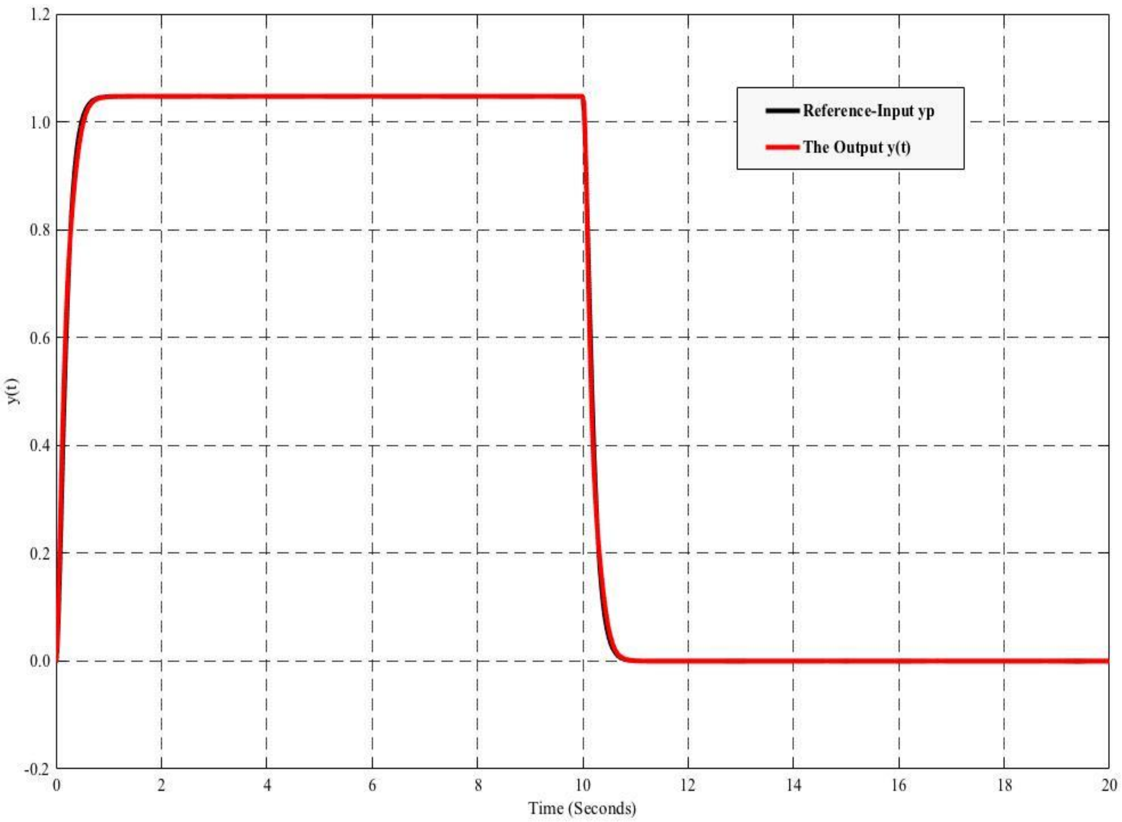



*Figure 3: The Output $y(t)$ tracks the reference input $y_p$ for problem which is addressed in example 1 with m=0.5, n=0.05, $k_1$ = 22 and $k_2$ =256 using the input-output feedback linearization technique.*

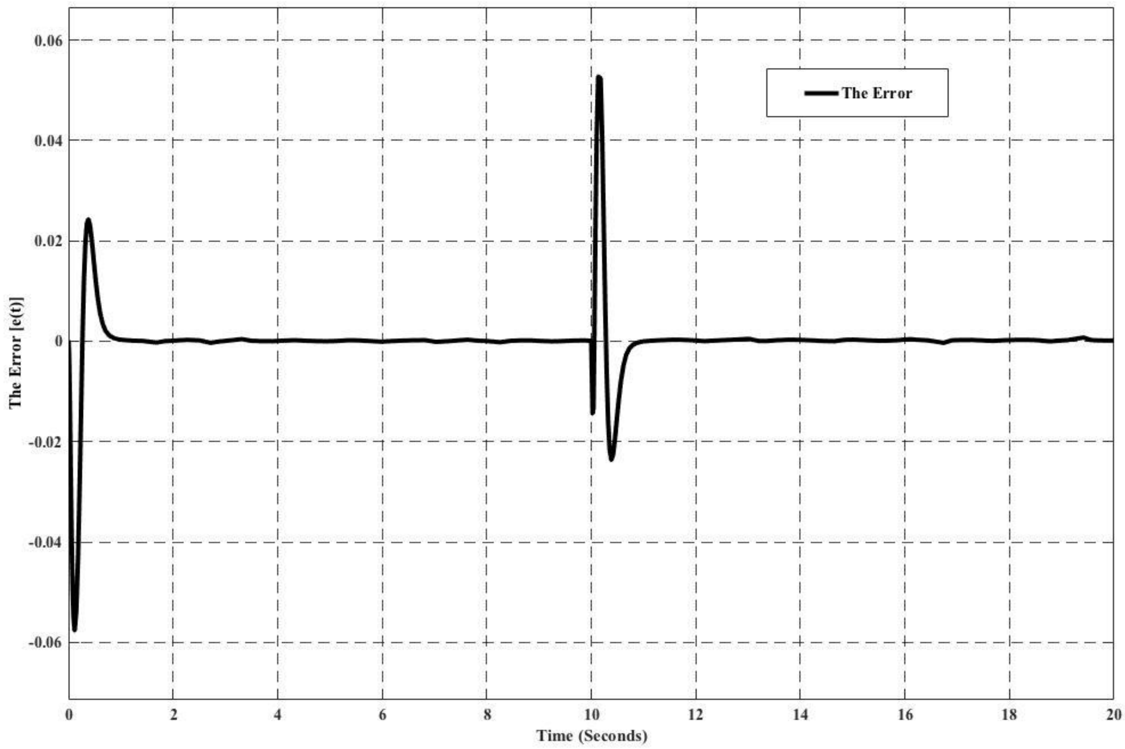



*Figure 4: The error for problem which addressed in example 1 with m=0.5, n=0.05, k1 = 22 and $k_2$ =256 using the input-output feedback linearization technique.*

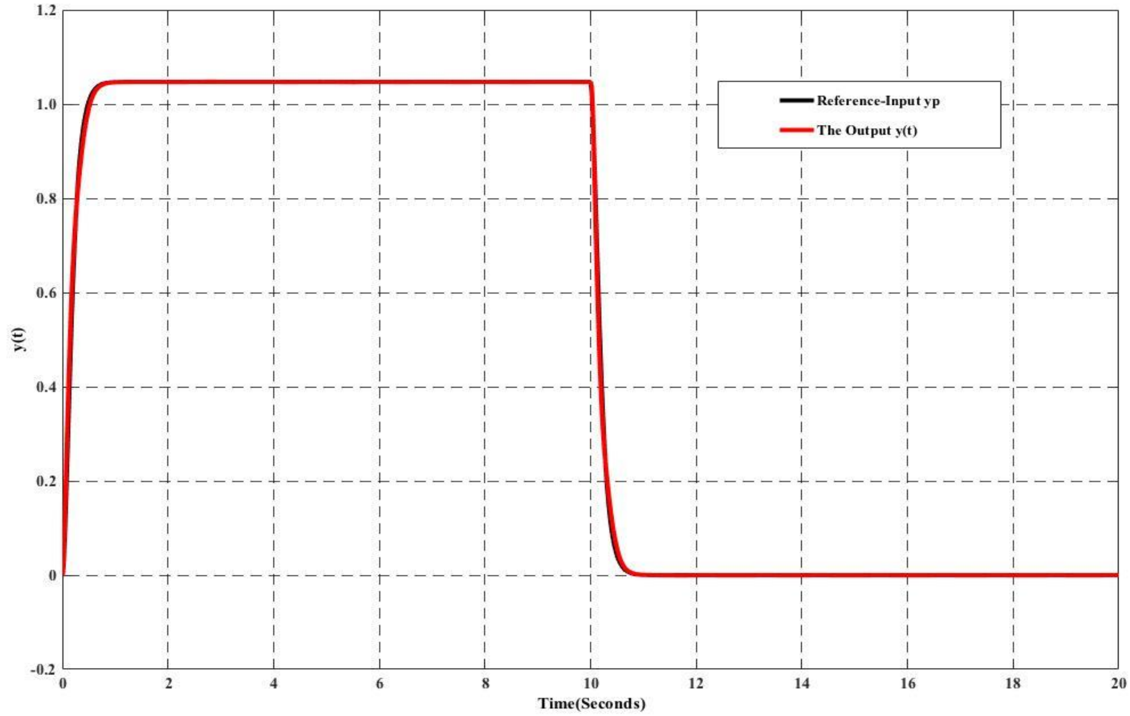



*Figure 5: The output $y(t)$ tracks the reference input $y_p$for problem which addressed in example 2 when 2 with m=0.5, n=0.05, k1 = 22 and $k_2$ =256 using the input-output feedback linearization technique*

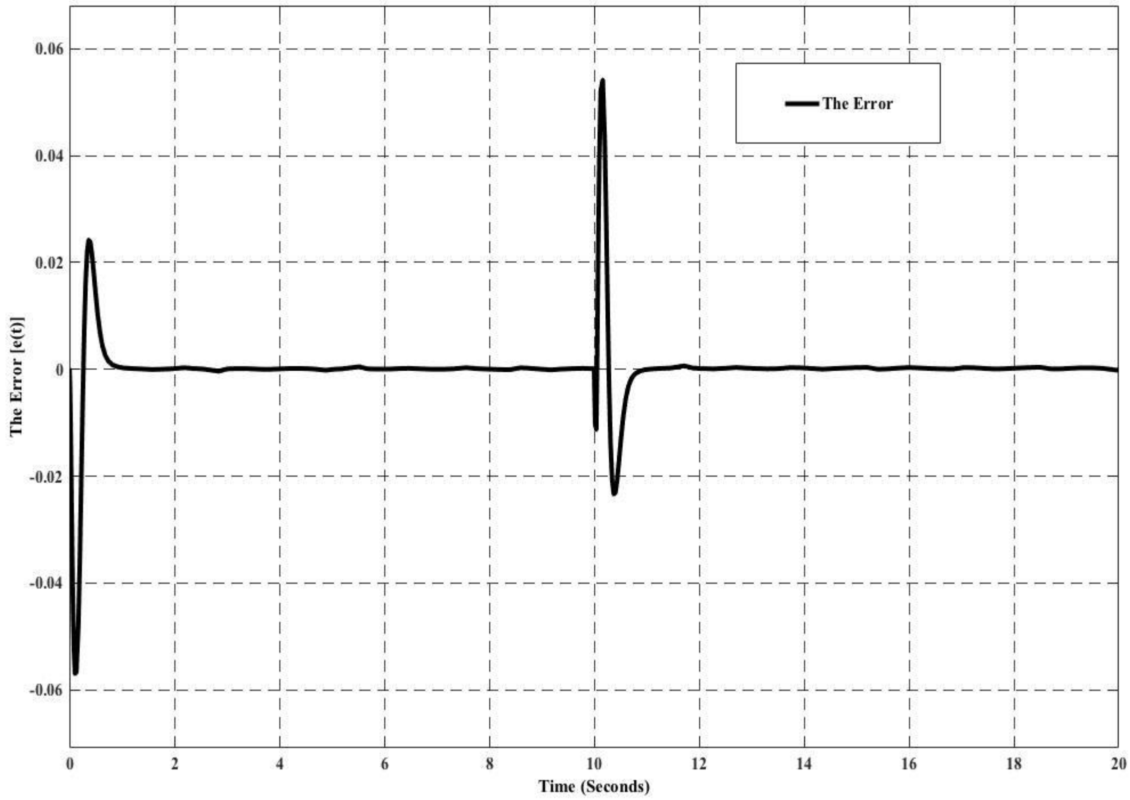



*Figure 6: The error for problem which addressed in example 2 with $m = 0.5, n = 0.05, k_1 = 22$ and $k_2$ =256 using the input-output feedback linearization technique.*

Based on the obtained results, the error observed in both cases falls within the range of $-0.02$ to $0.05$. However, in order to further enhance these results, an adjustment will be made by increasing the values of variables $k_1$ and $k_2$ by 65 and 725 respectively. By utilizing the new values of $k_1$ and $k_2$, improved outcomes are achieved, as illustrated in the subsequent figures. This modification aims to refine the performance and accuracy of the system, potentially reducing the error and yielding more desirable results. Overall, the decision to increase $k_1$ and $k_2$ represents an attempt to optimize the system's behavior, striving for improved accuracy and performance. The subsequent figures provide visual evidence of the effectiveness of this adjustment, allowing for a clearer understanding of the system's enhanced capabilities and its ability to produce more desirable outcomes.

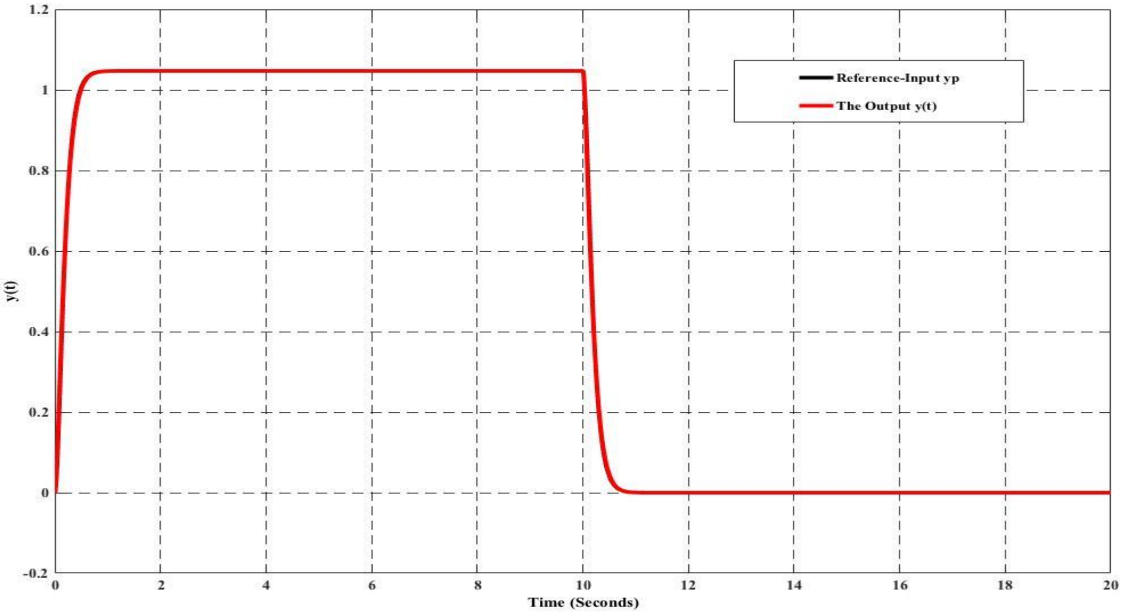



*Figure 7: The Output $y(t)$ Tracks the Reference Input $y_p$ for problem which addressed in example 1 when m=0.5, n=0.05, k1 = 65 and $k_2$ =725 using the input-output feedback linearization technique).*

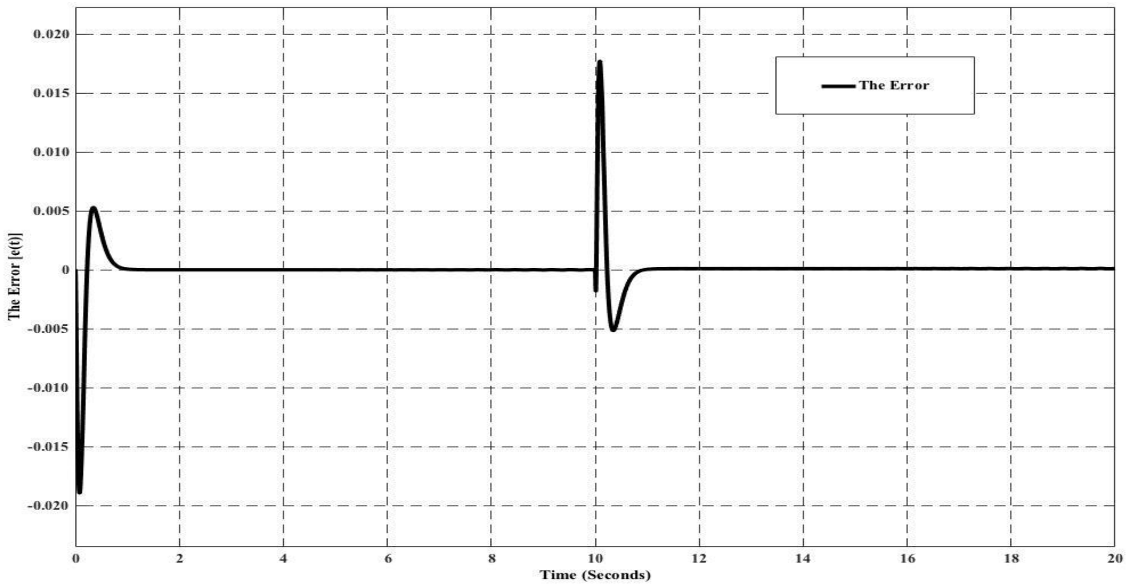



*Figure 8: The error for problem which addressed in example 1 with $m = 0.5, n = 0.05, k_1 = 65$ and $k_2 = 725$ using the input-output feedback linearization technique.*

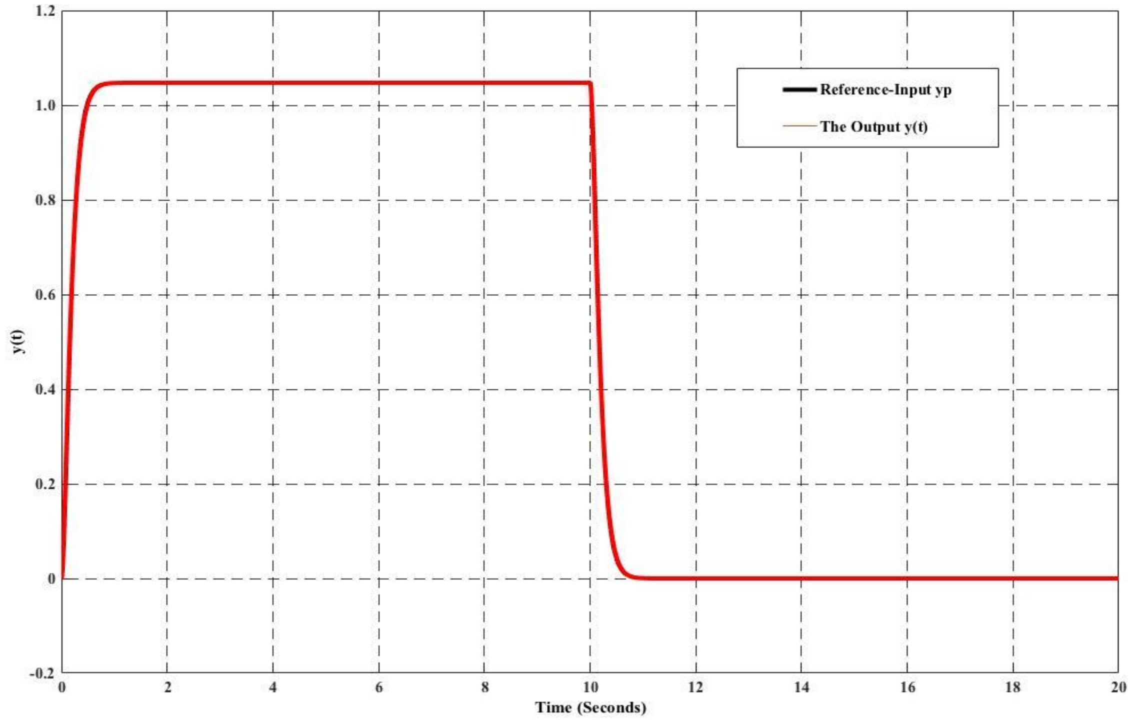



*Figure 9: The output y(t) tracks the reference input $y_p$ for problem which addressed in example 2 when m=0.5, n=0.05,* $k_1 = 65$ *and* $k_2$ *=725 using the input-output feedback linearization technique.*

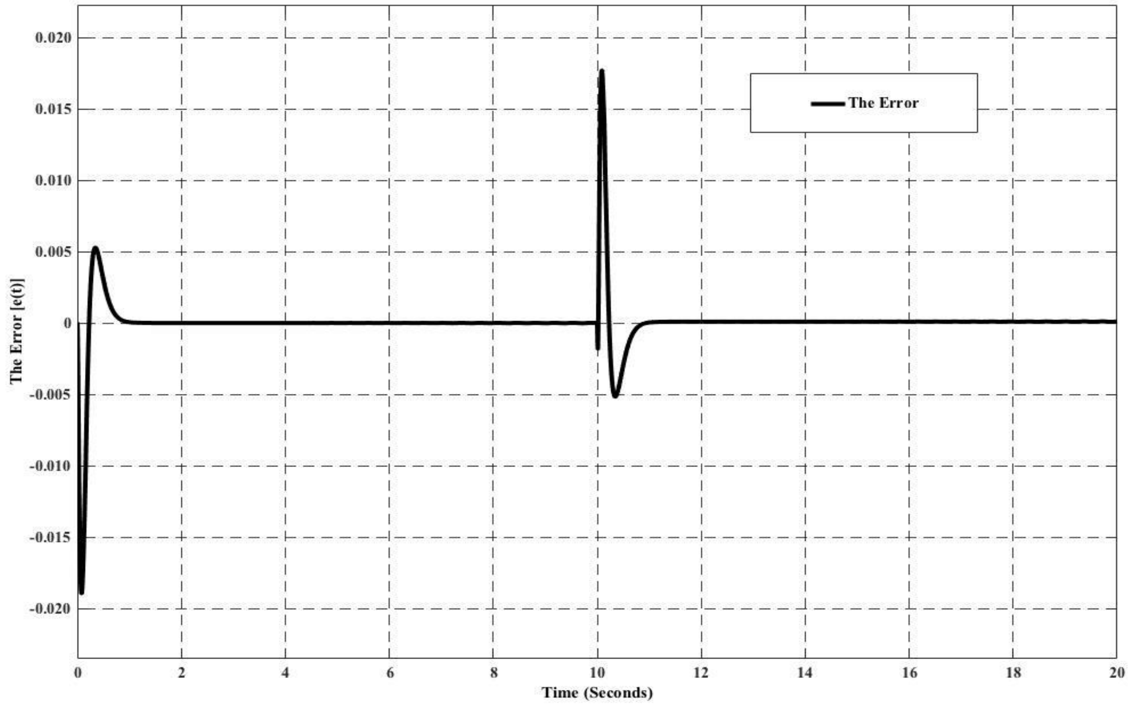



*Figure 10: The error for problem which addressed in example 2 with* $m = 0.5, n = 0.05, k_1 = 65$ *and* $k_2$ *=725 using the input-output feedback linearization technique*

From the results shown in Figures 7, 8, 9, and 10, a significant improvement has been achieved, and the error range has been decreased from $-0.02: 0.05$ to -0.005: 0.016. This reduction in the error range demonstrates the effectiveness of the applied modifications and optimizations in enhancing the accuracy and precision of the system. These findings indicate that the adjustments made have successfully fine-tuned the system's performance, leading to more reliable and desirable outcomes.

**Task (2)**

Simulation using the input-state feedback linearization technique. In Task (2), we use the same parameters that have been used in Task (1). The figures labeled as 11 and 12 display the simulation results obtained for the problem presented in Example 1. On the other hand, the figures denoted as 13 and 14 present the results obtained specifically for the problem addressed in Example 2.

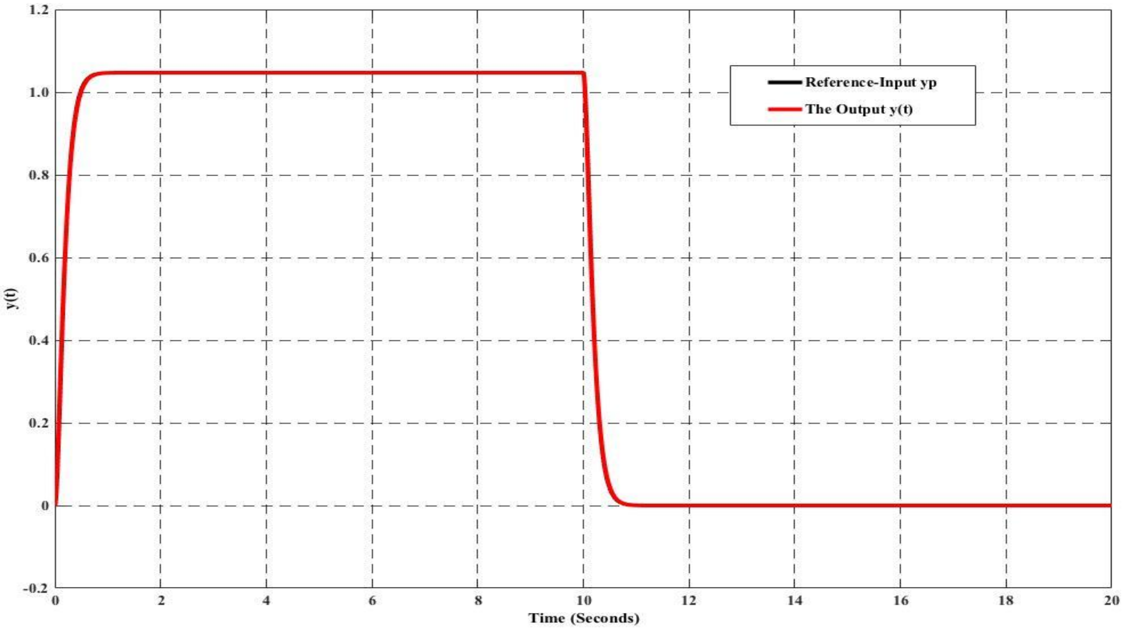



*Figure 11: The output $y(t)$ tracks the reference input $y_p$ for problem which is addressed in example 1 with $m = 0.5, n = 0.05$, $k_1 = 22$ and $k_2$ =256 using the input-state feedback linearization technique.*

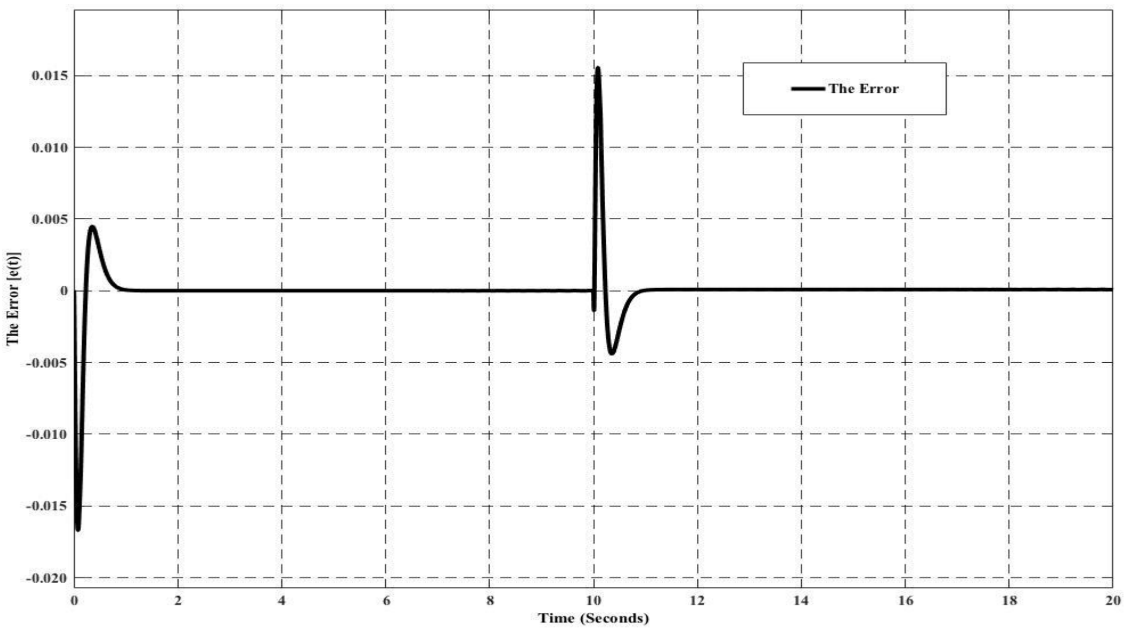



*Figure 12: The error for problem which addressed in example 1 with $m = 0.5, n = 0.05, k_1$= 22 and $k_2$ =256 using the input-state feedback linearization technique.*

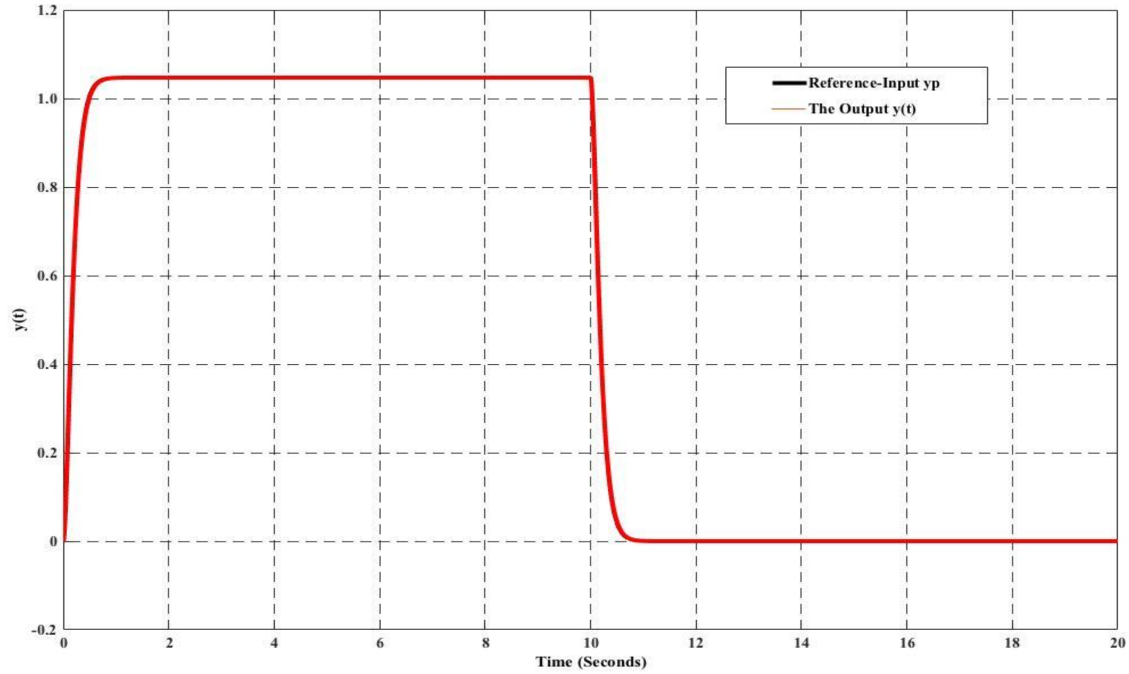



*Figure 13: The output y(t) tracks the reference input $y_p$ for problem which is addressed in example 2 with m=0.5, n=0.05, k1 = 22 and $k_2$ =256 using the input-state feedback linearization technique*

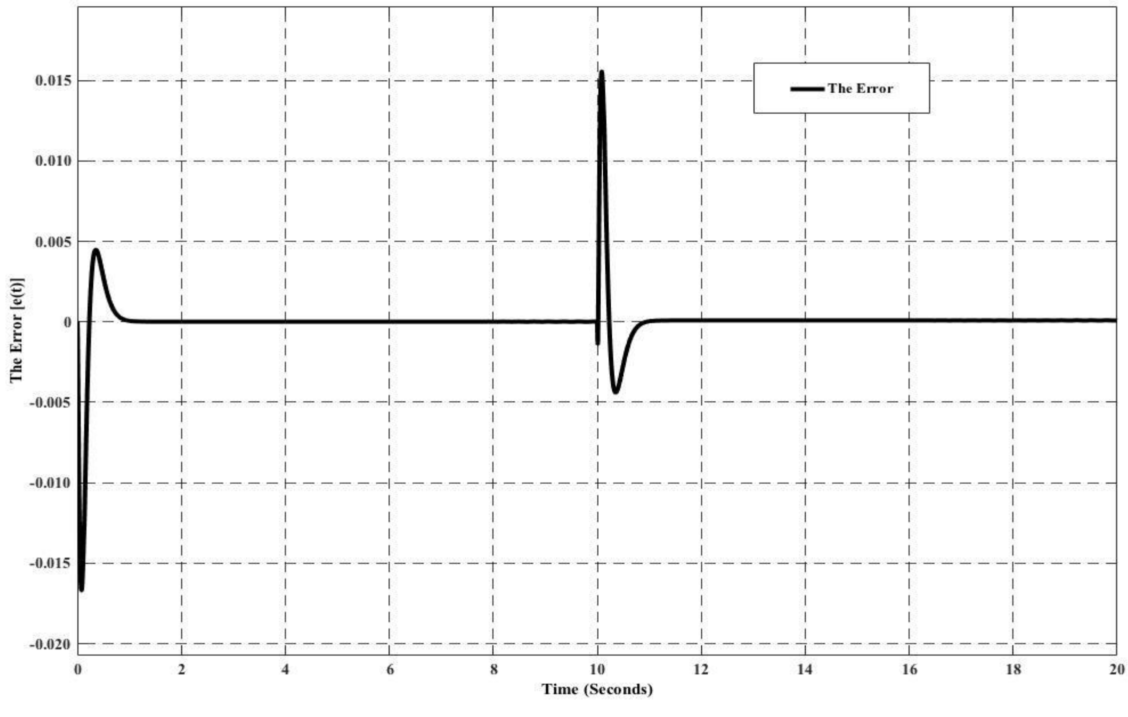



*Figure 14: The error for problem which addressed in example 2 with $m = 0.5, n = 0.05$, $k_1 = 22$ and $k_2$ =256 using the input-state feedback linearization technique .*

To reduce the error and enhance the obtained results, the values of $k_1$ and $k_2$ will be adjusted, by increasing them to 65 and 725 respectively. Figures 15 and 16 illustrate the results achieved for the problem described in Example 1. Similarly, Figures 17 and 18 illustrate the results obtained for the problem described in Example 2.

As anticipated, notable improvements have been accomplished, leading to a reduction in the error range for both problems outlined in Example 1 and Example 2.

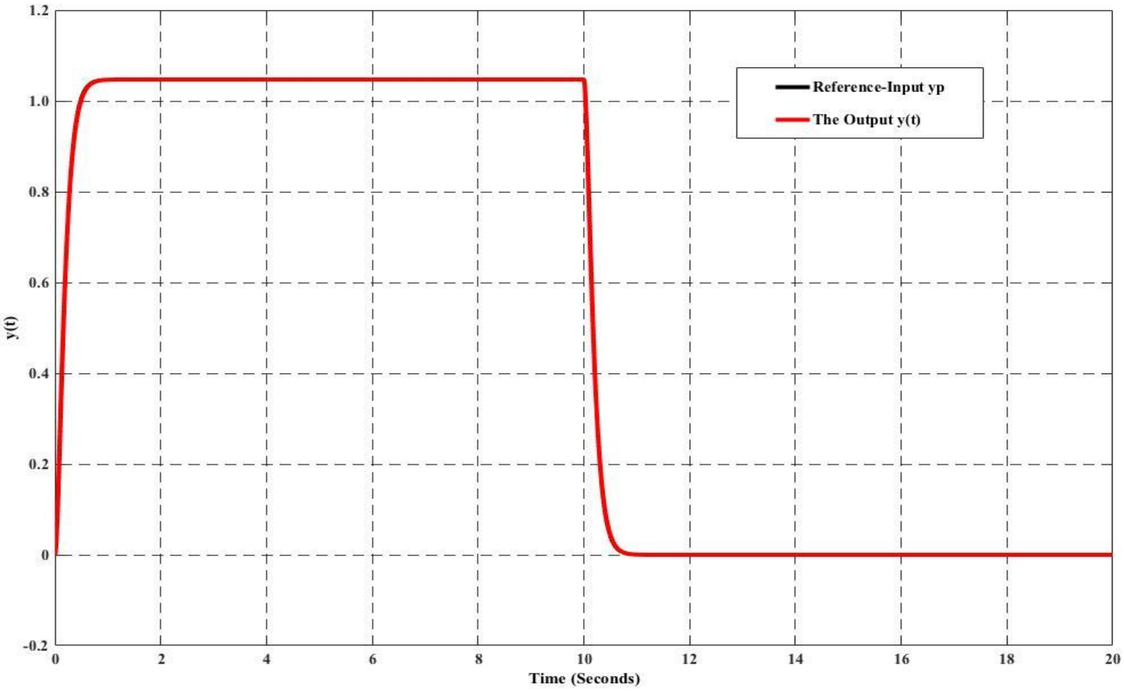



*Figure 15: The output $y(t)$ tracks the reference input $y_p$ for problem which is addressed in example 1 with $m = 0.5, n = 0.05, k_1 = 65$ and $k_2 = 725$ using the input-state feedback linearization technique.*

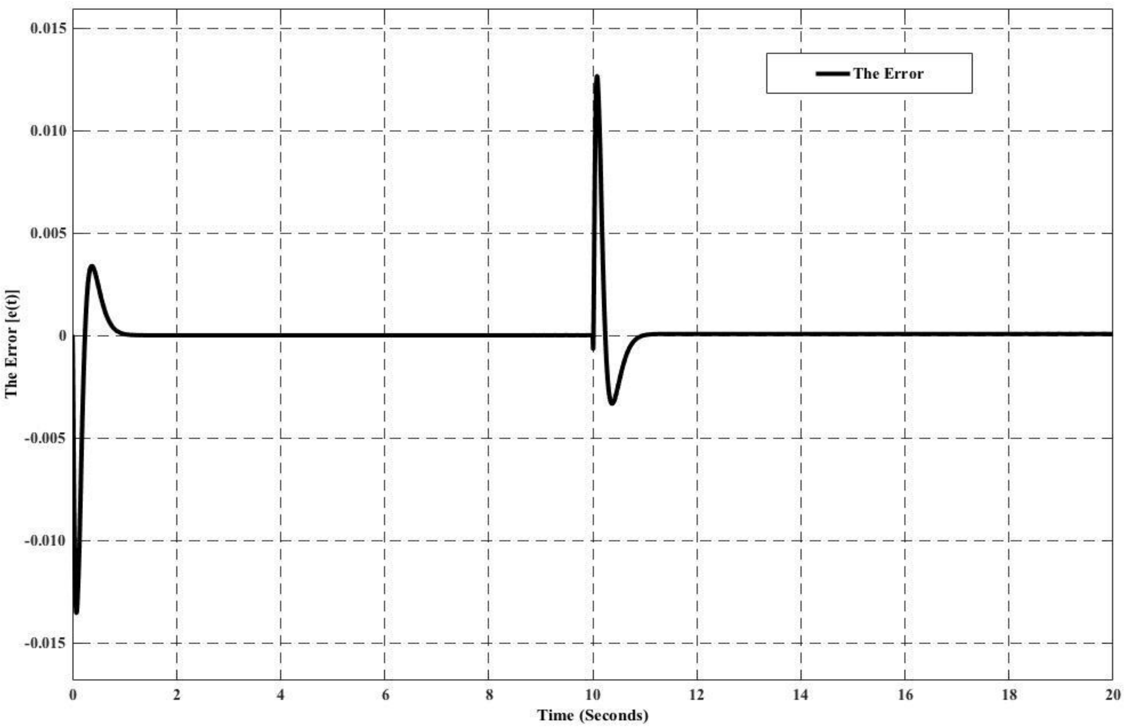



*Figure 16: The error for problem which addressed in example 1with $m = 0.5$, $n = 0.05, k_1 = 65$ and $k_2$ =725 using the input-state feedback linearization technique.*

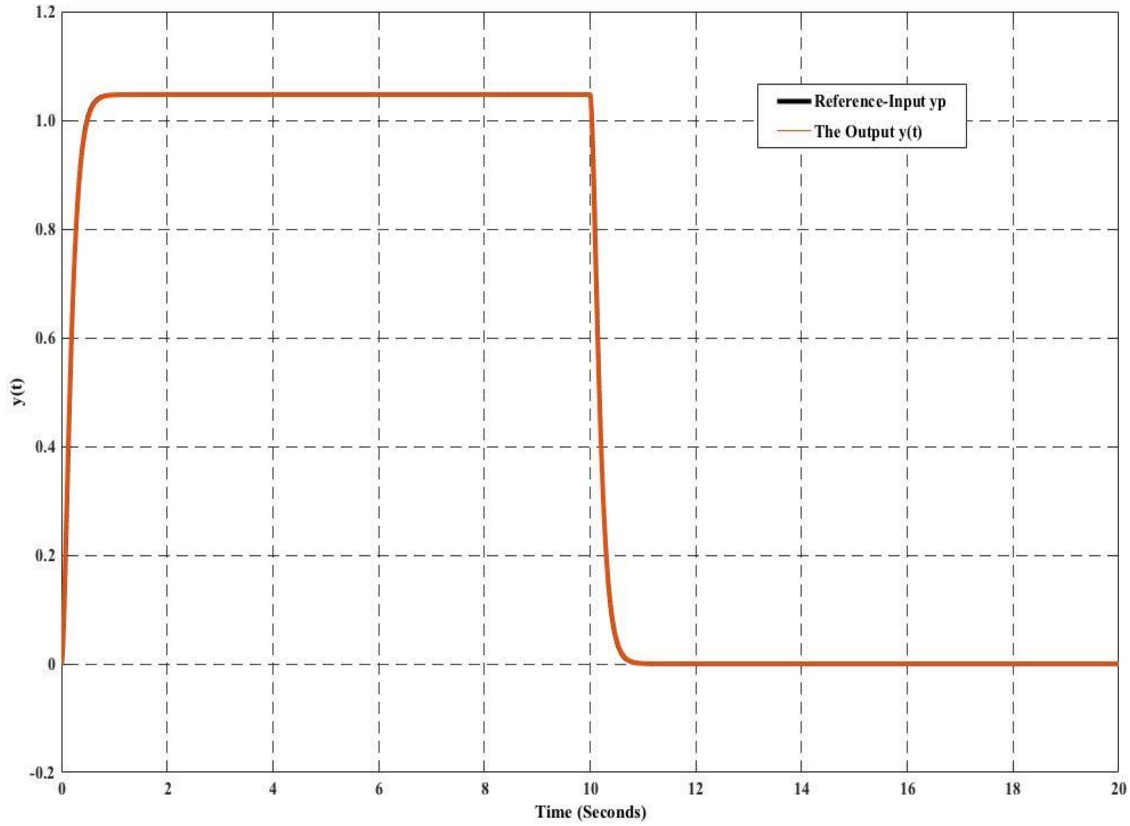



*Figure 17: The output $y(t)$ tracks the reference input $y_p$ for problem which is addressed in example 2 with $m = 0.5, n = 0.05$, $k_1$ = 65 and $k_2$ = 725 using the input-state feedback linearization technique.*

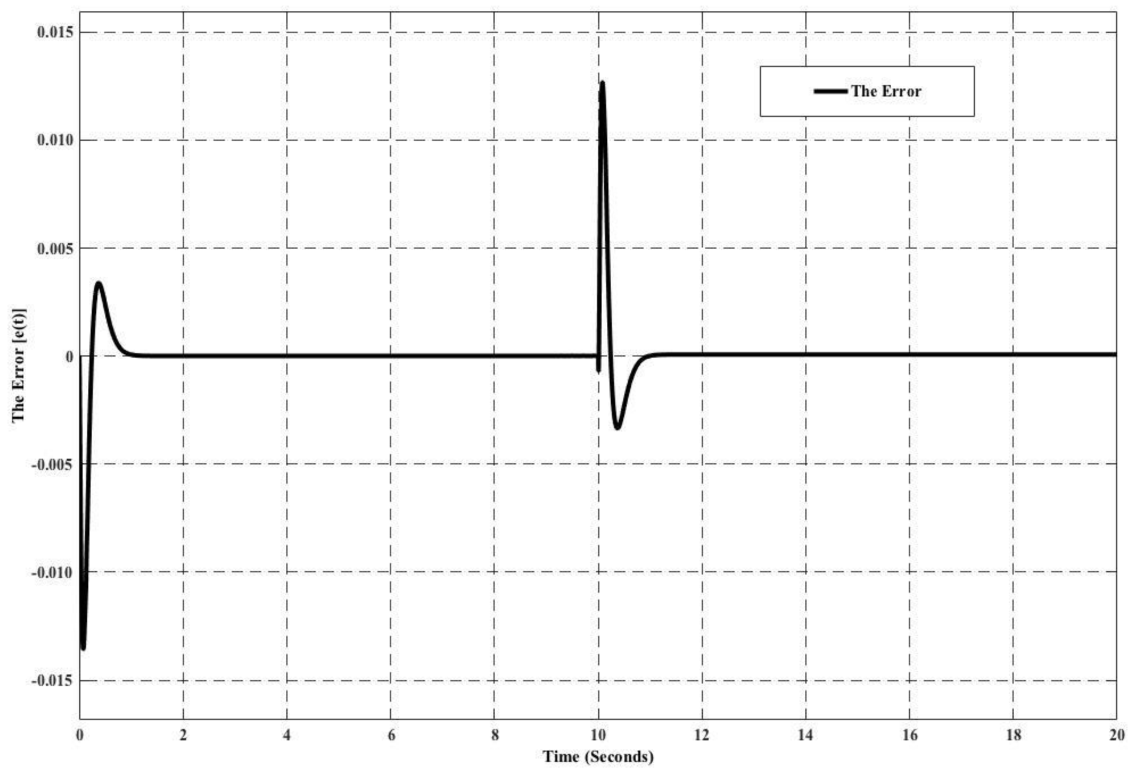



*Figure 18: The error for problem which addressed in example 2 with $m = 0.5, n = 0.05, k_1 = 65$ and $k_2$ =725 using the input-state feedback linearization technique.*

## 5. Conclusions and Future Work

In summary, input-state feedback linearization focuses on transforming the system dynamics by manipulating the internal state variables, while input-output feedback linearization achieves linearization through manipulation of the input and output variables. The choice between these approaches depends on the availability and ease of measurement of the system's internal states and the desired control objectives. Both techniques present good and close results for the problems under study described in Examples 1 and 2. Based on the results obtained, input-state feedback produces more accurate results compared with the input-output feedback linearization technique. This is because the applications used are known for direct measurement. Generally, input-state feedback linearization is more suitable than input-output feedback linearization in certain practical applications. Where direct measurement or reliable estimation of the system's internal states is feasible.

In future work, we propose exploring the challenges associated with linearizing MIMO nonlinear systems. This would involve applying the principles of input output and input-state linearization techniques to address the complexities of MIMO systems. By investigating these techniques, we can gain insights into effectively controlling and analyzing the behavior of MIMO nonlinear systems. Additionally, we suggest delving into the concept of linearization in the context of nonlinear adaptive systems, where the characteristics and parameters of the systems are either completely or partially unknown. This presents a unique set of challenges that require specialized approaches for achieving linearization and control. By examining the linearization concept within the framework of nonlinear adaptive systems, we can further enhance the understanding and develop effective strategies to handle such systems.

## 6. References

[1] Henson, Michael A., and Dale E. Seborg. ”Feedback linearizing control.” Nonlinear process control. Vol. 4. Upper Saddle River, NJ, USA: Prentice-Hall, 1997.

[2] Marquez, Horacio J. Nonlinear control systems: analysis and design. Vol. 161. Hoboken, NJ, USA: John Wiley, 2003.

[3] Khalil, Hassan K. ”Adaptive output feedback control of nonlinear systems represented by input-output models.” IEEE Transactions on Automatic Control 41.2 (1996): 177-188.

[4] H. K. Khalil and J. W. Grizzle, Nonlinear systems. Prentice hall Upper Saddle River, NJ, 2002, vol. 3.

[5] Wang, Jianliang, and W. Rugh. ”Feedback linearization families for nonlinear systems.” IEEE Transactions on Automatic Control 32.10 (1987): 935-940.

[6] Sastry, Shankar. Nonlinear systems: analysis, stability, and control. Vol.10. Springer Science and Business Media, 2013.

[7] Ghozlane, Wafa, and Jilani Knani. ”Nonlinear control of MIMO system using feedback linearization control method and PD controller for tracking purpose.” 5th International Conference on Control Engineering and Information Technology. Proceeding of Engineering and Technology. Vol. 32. 2017.

[8] Petlenkov, E. ”NN-ANARX structure based dynamic output feedback linearization for control of nonlinear MIMO systems.” 2007 Mediterranean Conference on Control and Automation. IEEE, 2007.

[9] Li, Zheng, et al. ”Nonlinear decoupling control of two-terminal MMCHVDC based on feedback linearization.” IEEE Transactions on Power Delivery 34.1 (2018): 376-386.

[10] Yang, Shunfeng, Peng Wang, and Yi Tang. ”Feedback linearizationbased current control strategy for modular multilevel converters.” IEEE Transactions on Power Electronics 33.1 (2017): 161-174.

[11] Chen, Dengyi, and Xiaocun Hu. ”A Multi-objective Feedback Linearization Control of a Cuk Converter.” IEEE Journal of Emerging and Selected Topics in Power Electronics (2023).

[12] Abdusamad, Abdunaser, and Mohamed Aburakhis. ”Adaptive Control of Nonlinear Systems Represented by Extreme Learning Machine (ELM) and the Fuzzy Logic Control (FLC).” 2021 IEEE 1st International Maghreb Meeting of the Conference on Sciences and Techniques of Automatic Control and Computer Engineering MI-STA. IEEE, 2021.

[13] Abdusamad, Abdunaser, and Mohamed Aburakhis. ”Tracking Control of Adaptive MIMO Nonlinear Systems Using Fuzzy Logic Control and Extreme Learning Machine.” 2023 IEEE 3rd International Maghreb Meeting of the Conference on Sciences and Techniques of Automatic Control and Computer Engineering (MI-STA). IEEE, 2023.

[14] Wu, Yuanqing, et al. ”Performance recovery of dynamic feedback linearization methods for multivariable nonlinear systems.” IEEE Transactions on Automatic Control 65.4 (2019): 1365-1380.

[15] Belikov, Juri, and Eduard Petlenkov. ”Model reference control of nonlinear TITO systems by dynamic output feedback linearization of neural network based ANARX models.” 2009 IEEE Control Applications,(CCA) and Intelligent Control,(ISIC). IEEE, 2009.

[16] Martinez-Trevio, Blanca Areli, et al. ”Nonlinear control for output voltage regulation of a boost converter with a constant power load.” IEEE Transactions on Power Electronics 34.11 (2019): 10381-10385.

[17] Liptk, Tom, et al. ”Input-state linearization of mechanical system.” American Journal of Mechanical Engineering 5.6 (2017): 298-302.