SJST
sjst.scst.edu.ly
Surman **J**ournal for **S**cience and **T**echnology
ISSN: Online (2790-5721) - Print (2790-5713)
مجلة صرمان للعلوم والتقنية
Vol **7**, No.2.Jun – Dec. 2025
Pages: 048 ~ 057

Digital Filters in terms of Frequency and impulse response of ideal low-pass, ideal high-pass, band-pass and band-stop filters using inverse (DTFT) Definition

Imad Omara Shebani Etomi[1] , Zakariya Ali Saeid[2] , Mustfa Salem Eler[3], Salah Eddin Altaher Albakoush[4] , Adel Ali Ramadan Gamati[5]

Surman College of Science and Technology- Surman, Libya

ima_etomi@scst.edu.ly[*1] , *zakariya@scst.edu.ly*[2] , *mustf_salem@scst.edu.ly*[3] , *sartmar@scst.edu.ly*[4] , *adel_gmati@scst.edu.ly*[5]

الملخص

في هذه الورقة يتم دراسة أربعة أنواع من المرشحات الرقمية من حيث الترددات وكيفية ايجاد استجابة النبضة لمرشحات التمرير المنخفض المثالية، والتمرير العالي المثالي، وتمرير النطاق، وإيقاف النطاق باستخدام تعريف DTFT العكسي. كما يُظهر برنامج Matlab استجابة النبضة لأنواع المرشحات الأربعة من خلال رسم استجابة تردد السعة, و تحديد القيم المطلقة لتحويل فورييه السريع لكل استجابة نبضة من المرشحات وسوف يتم توليد إشارة عشوائية كمدخل لأنواع المرشحات الأربعة وتم رسم استجابة تردد السعة للخرج حيث تم تطبيق مقطع كمدخل لأنواع المرشحات الأربعة وتم رسم استجابة التردد للخرج.

Abstract:
The basic principles of this paper about the four types of digital filters are studied in terms of frequencies and how to find the impulse response of ideal low-pass, ideal high-pass, band-pass, and band-stop filters using the inverse DTFT definition. Moreover, the Matlab program displays the impulse response of the four filter types by plotting the amplitude-frequency response and determining the absolute values of the fast Fourier transform for each filter impulse response. A random signal will be generated as an input to the four filter types, and the amplitude-frequency response of the output will be plotted. A segment was applied as an input to the four filter types, and the output frequency response was plotted.

Key words: Definition of the inverse (DTFT), Ideal low pass, Ideal high pass, Cut-off frequency, Filters impulse response, Random signal, Frequency response, Rectangular filters H(ω)

Submitted: 19/07/2025 *Accepted: 25/08/2025*

1. Introduction:

In this papers it shows four types of digital filters in terms of frequencies and finds the impulse response for ideal low pass, ideal high pass, band pass, and band stop filters by using the definition of the inverse DTFT. Moreover, the Matlab program shows the impulse response for the four types of filters by plotting the Amplitude frequency response. Also, it shows absolute values of the FFT's of each of the filter's impulse response. In addition, it introduces the Blackman window to give an idea how it changes the characteristic of the filters. In the second part of these papers, it has been generated a random signal as an input to the four types of filters and amplitude frequency response of the output were plotted.

2. work and Background:

First, it has been calculated the impulse response for ideal low pass, high pass, band pass and band stop filters.

3. The ideal law pass filter:

Inverse DTFT:-

$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{jw}).e^{jwn}dw \qquad \text{-------------- (1)}$$

$$= \frac{1}{2\pi}\left[\int_{-wc}^{wc} e^{jwn}dw\right]$$

$$\frac{1}{2\pi}\left[\left(\frac{e^{jwcn} - e^{-jwcn}}{2j}\right)\right]$$

$$\because \sin x\left(\frac{e^{jx} - e^{-jx}}{2j}\right) \qquad \text{----------------(2)}$$

$$\therefore h[n] = \frac{1}{\pi n}(\sin wcn) \qquad \text{----------------(3)}$$

$$\lim_{n\to 0}\left(\frac{\sin(wcn)}{\pi n}\right) = \frac{wc\cos(wcn)}{\pi} = \frac{wc}{\pi} \qquad \text{----------------(4)}$$

$$h[n]_{LPF} = \begin{cases} \frac{wc}{\pi} & ; \quad n = 0 \\ \frac{\sin(wcn)}{n\pi} & ; n \neq 0 \end{cases} \qquad \text{----------------(5)}$$

4. The ideal High pass filter:

Inverse DTFT:-

$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{jw}).e^{jwn}dw \qquad \text{----------------(6)}$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-wc} e^{jwn}dw + \int_{wc}^{\pi} e^{jwn}dw\right]$$

$$= \frac{1}{2\pi}\left[\frac{e^{jwn}}{jn}\Big| \begin{matrix}-wc \\ -\pi\end{matrix} + \frac{e^{jwn}}{jn}\Big| \begin{matrix}\pi \\ wc\end{matrix}\right] = \frac{1}{2\pi jn}\left[e^{-jwcn} - e^{-j\pi n} + e^{j\pi n} - e^{jwcn}\right]$$

$$= \frac{1}{2\pi}\left[\left(\frac{-e^{jwcn} + e^{-jwcn}}{2j}\right) + \left(\frac{e^{j\pi n} + e^{-j\pi n}}{2j}\right)\right]$$

$$= \frac{1}{2\pi}[-\sin wcn + \sin \pi n] = \frac{1}{\pi n}[\sin \pi n + \sin wcn]$$

$\because \frac{\sin \pi n}{\pi n} = \delta[n]$ ----------------(7)

$\therefore h[n] = \frac{1}{\pi n}[\delta[n] - \sin wcn]$ ----------------(8)

So, $h[n]_{HP} = \delta[n] - h[n]_{LP}$ ----------------(9)

$$h[n]_{HP} = \begin{cases} \frac{-wc}{\pi} + 1 & ; \ n = 0 \\ \frac{-\sin wcn}{\pi n} & ; n \neq 0 \end{cases}$$ ----------------(10)

5. The correlation between the low pass and high pass filter is:

$h[n]_{HP} = \delta[n] - h[n]_{LP}$ or $H(w)_{HP} = 1 - H(w)_{LP}$ ----------------(11)

6. The ideal Band pass filter:

Inverse DTFT:-

$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{jw}).e^{jwn}dw$ ----------------(12)

$$= \frac{1}{2\pi}\left[\int_{-wb}^{-wa} e^{jwn}dw + \int_{wa}^{wb} e^{jwn}dw\right]$$

$$= \frac{1}{2\pi jn}\left[e^{-jwan} - e^{-jwbn} + e^{jwbn} - e^{jwan}\right]$$

$$= \frac{1}{2\pi}\left[\left(\frac{e^{jwbn} - e^{-jwbn}}{2j}\right) - \left(\frac{e^{jwan} - e^{-jwan}}{2j}\right)\right]$$

$$= \frac{1}{\pi n}[\sin wbn - \sin wan]$$

$$h[n]_{BPF} = \begin{cases} \frac{1}{\pi n}(\sin wbn - \sin wan) & ; \ n \neq 0 \\ \frac{1}{\pi}(wb - wa) & ; n = 0 \end{cases}$$ ----------------(13)

7. The ideal Band stop filter:

Inverse DTFT:-

$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{jw}).e^{jwn}dw$ ----------------(14)

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-wb} e^{jwn}dw + \int_{-wa}^{wa} e^{jwn}dw + \int_{wb}^{\pi} e^{jwn}dw\right]$$

$$\frac{1}{2\pi jn}\left[e^{-jwbn} - e^{-j\pi n} + e^{jwan} - e^{-jwan} + e^{j\pi n} - e^{jwbn}\right]$$

$$= \frac{1}{2\pi}\left[\left(\frac{e^{jwan} - e^{-jwan}}{2j}\right) - \left(\frac{e^{jwbn} - e^{-jwbn}}{2j}\right) + \left(\frac{e^{j\pi n} - e^{-j\pi n}}{2j}\right)\right]$$

$$= \frac{1}{\pi n}(\sin wan - \sin wbn + \sin \pi n)$$

$$\because \frac{\sin \pi n}{\pi n} = \delta[n] \quad \text{----------------(15)}$$

$$\therefore h[n] = \frac{-1}{\pi n}(\sin wbn - \sin wan) + \delta[n] \quad \text{----------------(16)}$$

$$h[n]_{BSF} = \delta[n] - h[n]_{BPF}$$

$$h[n]_{BSF} = \begin{cases} \frac{-1}{\pi}(wb - wa) + 1 & ; n = 0 \\ \frac{-1}{\pi n}(\sin wbn - \sin wan) & ;\ n \neq 0 \end{cases} \quad \text{----------------(17)}$$

8. The correlation between the band pass and stop pass filter is:

$h[n]_{BSF} = \delta[n] - h[n]_{BPF}$ or $H(w)_{BSF} = 1 - H(w)_{BPF}$ ----------------(18)

Second, it has been used the impulse response formula that derived in the first part for each filter and using MATLAB to truncate the filter impulse response using 31 samples {n=[-15,15]}. The cut-off frequency that it used in filters are; π/4 for the LPF and HPF and (π/4,π/2) for the BPF and BSF. The impulse response of each filter is shown in the following figure (1).

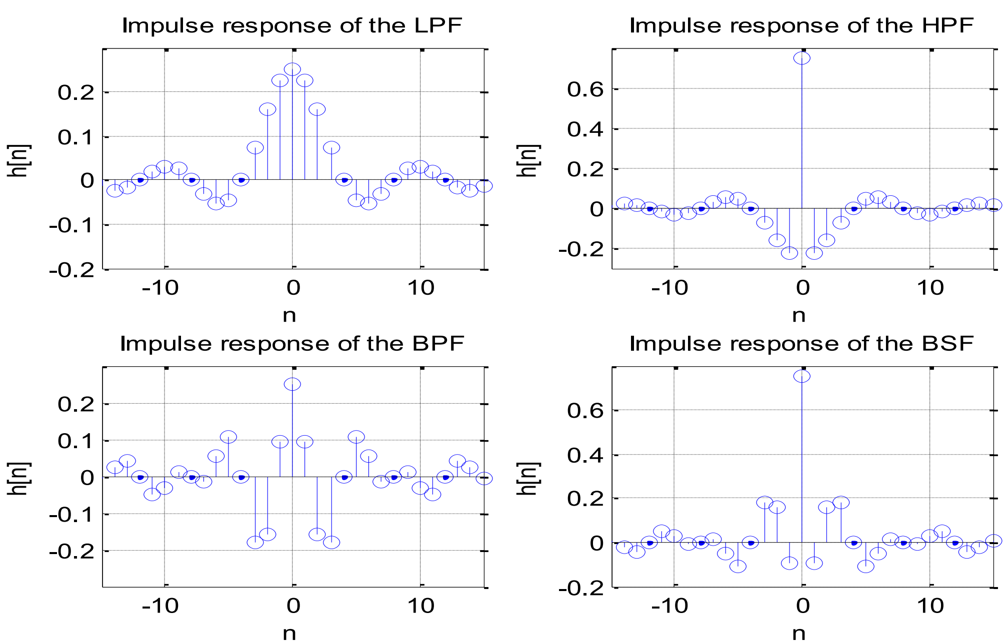

Fig.1. Impulse response of the four types of ideal digital filters.

Then, the FFT's each of the filter's impulse response is computed. Then the amplitude frequency response of the filters H(ω) is plotted as shown in figure(2).

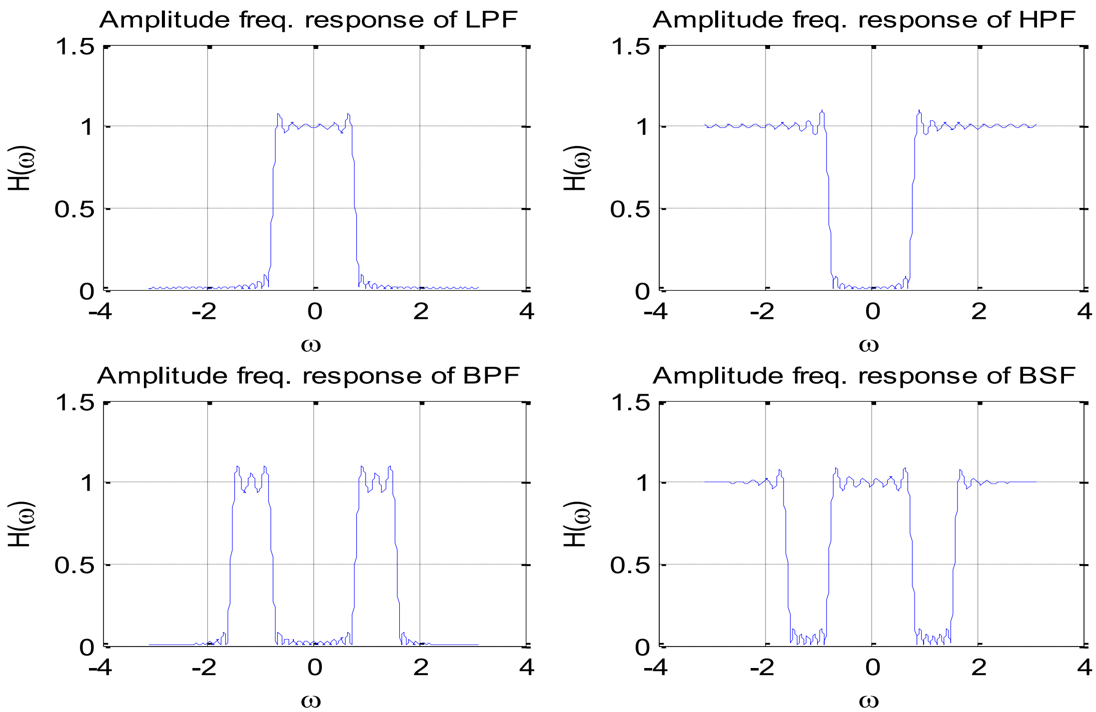

Fig.2. Amplitude frequency response of the four types of digital filters.

As it is clear from the four subfigures, the DTFT is defined from –pi to pi and the function fftshift() center the FFT about zero.

Also, the amplitude frequency response in dB for each filter is plotted as shown in figure (3). From this figure it can be can noted that the -3db (cut-off frequency) (half-power point) of the LPF and HPF is approximately corresponds to π/4. Also, for the BPF and BSF approximately corresponds to π/4 and π/2.

sjst.scst.edu.ly
Surman Journal for Science and Technology
ISSN: Online (2790-5721) - Print (2790-5713)
مجلة صرمان للعلوم والتقنية
Vol 7, No.2.Jun – Dec. 2025
Pages: 048 ~ 057

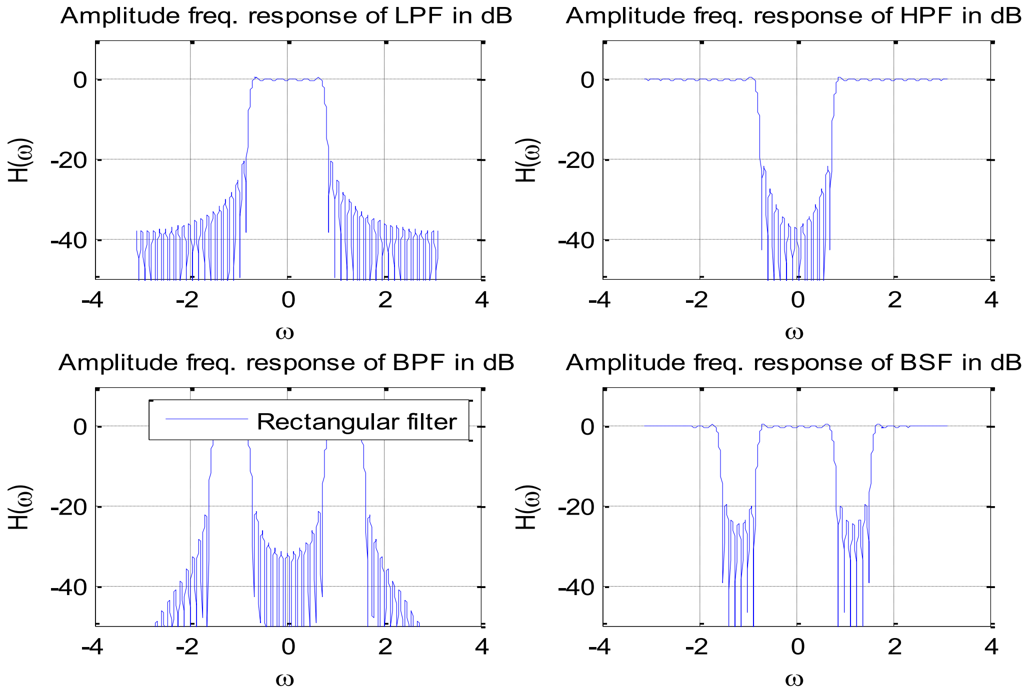

Fig.3. Amplitude frequency response in dB of the four types of ideal digital filters.

In this section, it is also used a windowing filter which is known as Blackman window filter and then the amplitude frequency response of Blackman window and rectangular filters H(ω) are plotted for the four types of filters as shown in figure (4).

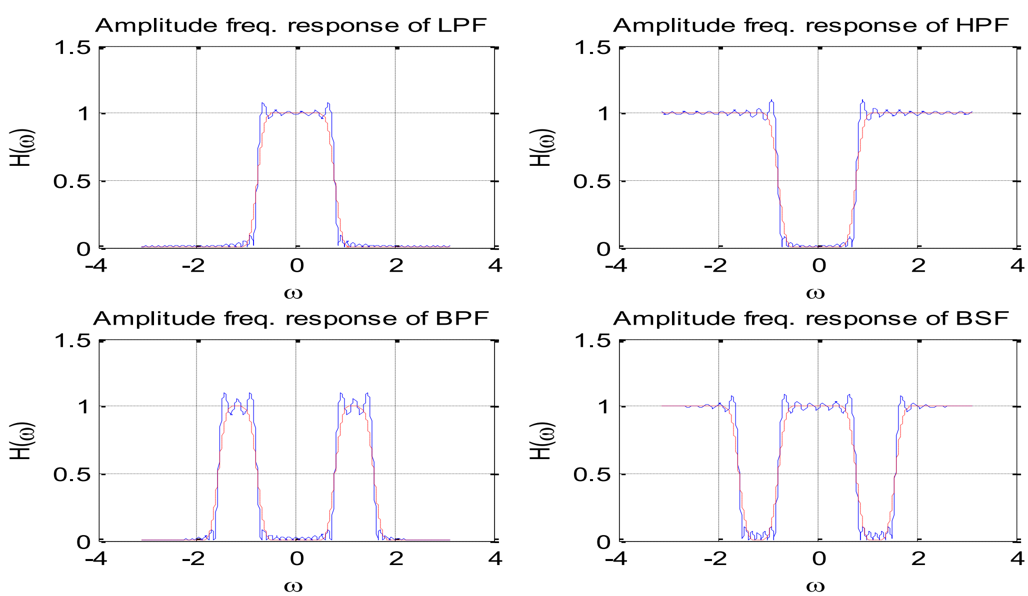

Fig.4. Amplitude frequency response of the four types rectangular filters compared with Blackman window filters

Surman Journal for Science and Technology
ISSN: Online (2790-5721) - Print (2790-5713)
مجلة صرمان للعلوم والتقنية
Vol 7, No.2.Jun – Dec. 2025
Pages: 048 ~ 057

In order to see clearly the differences between using rectangular and Black window filter, it is plotted the Amplitude frequency response of the four types in dB as shown in figure (5)

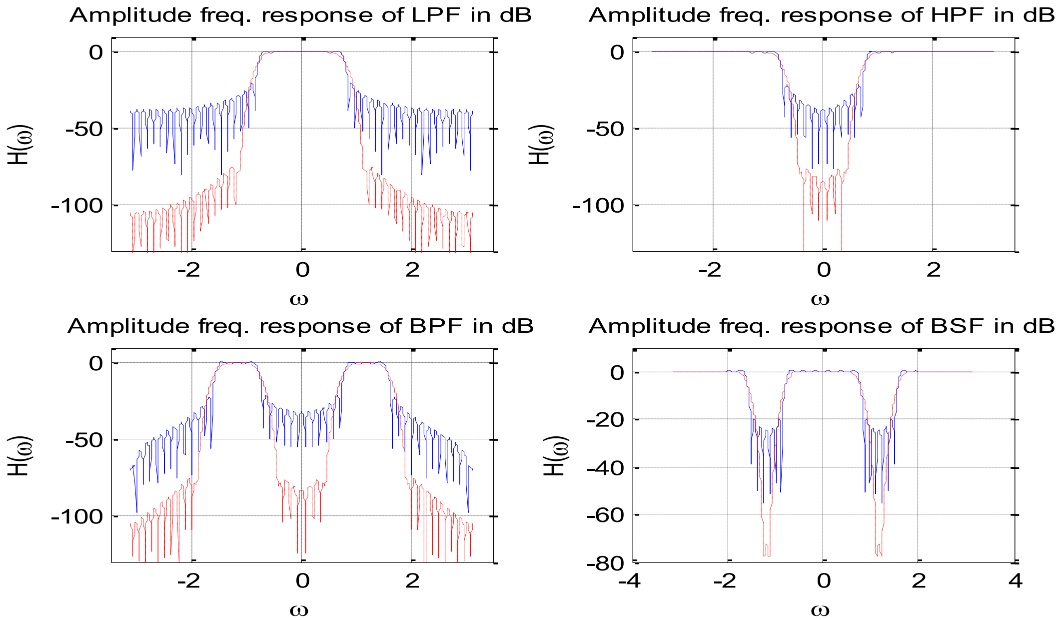

Fig.5. Amplitude frequency response of the four types Rectangular filters compared with Blackman window filters in dB.

It is obvious that using Blackman window gives better characteristics for filters in terms of stop band and pass band.

In the second part of the report, a random signal of size 1024 samples centered at zero is generated. Figure (6) shows the amplitude frequency response H(ω).

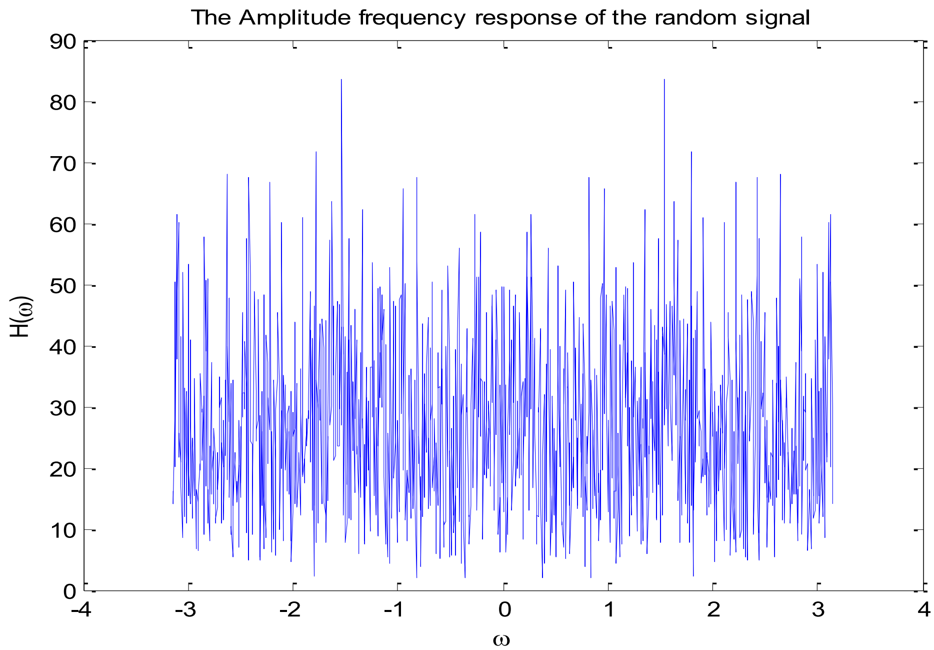

Fig.6. Amplitude Frequency Response of the Random signal

Then, the four filters applied to filter the random signal which it generated using convolution command. Then, the FFT of the filter impulse response is computed as shown in figure(7)

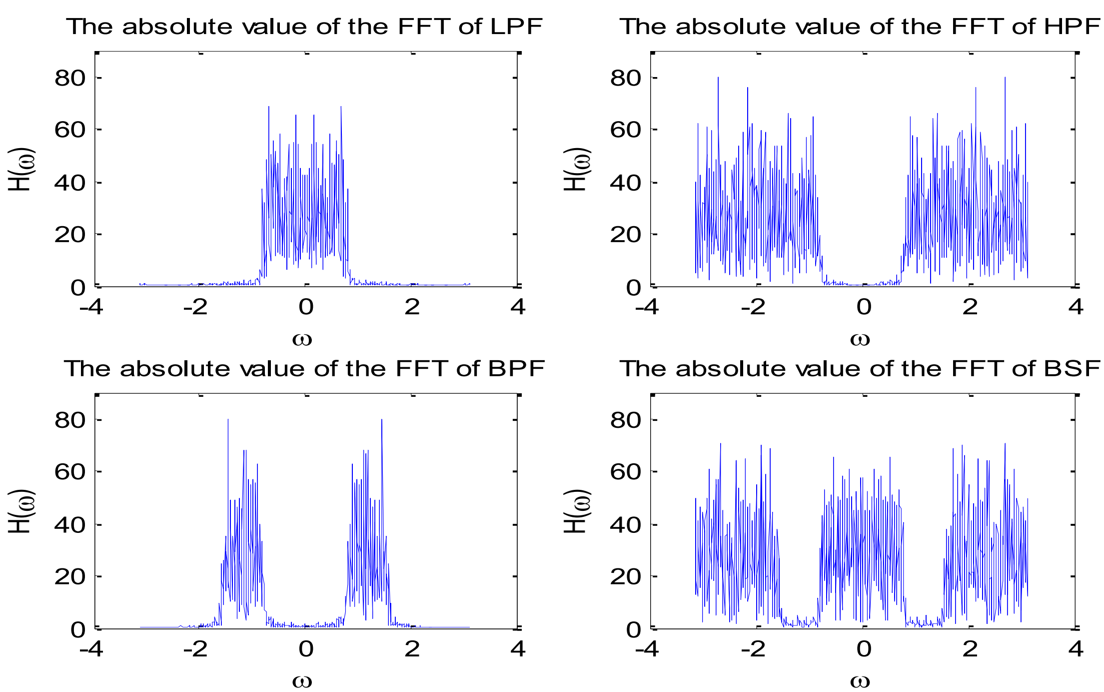

Fig.7. Absolute value of the FFT's of the four filters.

The last four subplots it makes sense, because in the first subplot the LPF allow to pass the frequencies from zero (DC component) up to certain frequency ω_c(cut-off frequency). In the second subplot the HPF pass the frequencies from ω_c(cut-off frequency) to π and attenuate the frequencies from zero to ω_c. In the third subplot, the BPF pass band of frequencies between ω_a and ω_b and reject other frequencies. In the last subplot, the BSF attenuate the frequencies between $\omega_a$and ω_b and allow others to pass. Finally, a clip to all types of filters are applied as shown in figure (8).

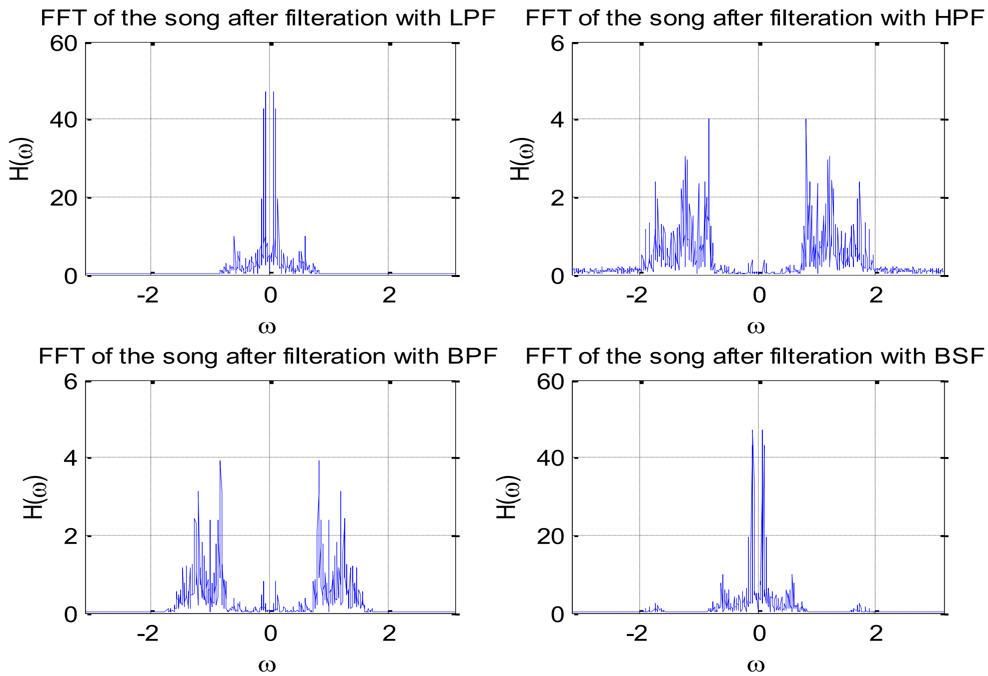

Fig.8. Absolute value of the FFT's of the filtered clip

9. Conclusion:

According to the results we got from this paper that the signal processing the function of a filter is to remove unwanted parts of the signal or to extract useful part of the signal. In these papers, it concentrates on the most popular filters (LPF, HPF, BPF, and BSF). In addition, it found that there are a correlation between LPF and HPF and also a correlation between BPF and BSF. From the results low-pass, high-pass, band-pass, and band-stop filters demonstrate useful characteristics. The process of filtering by the convolution of a signal with the impulse response of an ideal filter, and then obtaining the frequency response by taking the Fourier transform of the magnitude of that result are demonstrated in this study. Overall, the behaviors of four different types of ideal filters are learned from this filter study. Low-pass, high-pass, band-pass, and band-stop filters are all unique and useful in their own rights and also related by convolution, the Fourier transform, and as described in the section containing theoretical results. Using the impulse response of these filters and applying Blackman window, it shows how could filters are improved in terms of their characteristics. Moreover, it generates a random signal and applies to those filters and observes the result, it shows how those filters work such as some of them pass some frequencies while others reject other frequencies. Finally, it applies a clip to the four filters which shows how those filters affect on the clip by rejecting and passing some frequencies.

References:

1. J. O. Smith III, Introduction to Digital Filters with Audio Applications, Center for Computer Research in Music and Acoustics (CCRMA), Stanford University, September 2007 Edition.
2. Mitra, S. K. (1998). Digital Signal Processing: A Computer-Based Approach. New York, NY: McGraw-Hill.
3. Oppenheim, A. V.; Schafer, R. W. (1999). Discrete-Time Signal Processing. Upper Saddle River, NJ: Prentice-Hall. ISBN 9780137549207.
4. Kaiser, J .F. (1974). Nonrecursive Digital Filter Design Using the Io-sinh Window Function. Proc. 1974 IEEE Int. Symp. Circuit Theory. pp. 20–23.
5. Bergen, S. W. A.; Antoniou, A. (2005). "Design of Nonrecursive Digital Filters Using the Ultraspherical Window Function". EURASIP Journal on Applied Signal Processing. 2005 (12): 1910–1922. Bibcode:2005EJASP2005...44B. doi:10.1155/ASP.2005.1910.
6. Parks, T. W.; McClellan, J. H. (March 1972). "Chebyshev Approximation for Nonrecursive Digital Filters with Linear Phase". IEEE Trans. Circuit Theory. CT-19 (2): 189–194. doi:10.1109/TCT.1972.1083419.
7. Rabiner, L. R.; McClellan, J. H.; Parks, T. W. (April 1975). "FIR Digital Filter Design Techniques Using Weighted Chebyshev Approximation". Proc. IEEE. 63 (4): 595–610. Bibcode:1975IEEEP. 63..595R. doi:10.1109/PROC.1975.9794. S2CID 12579115