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Competence Amelioration of PMBLDC Motor using LQR- PID, Kalman Filter- PID and LQG Based on Kalman Filter-PID optimal Controllers for disturbance attenuation

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الملخص

في هذه الورقة البحثية ، تحليل الأداء ونمذجة ومحاكاة لمحرك التيار المستمر بدون فرش (PMBLDC) باستخدام وحدة التحكم الكلاسيكية (المتحكم التناسبي التفاضلي التكاملي PID) ووحدات التحكم المثلى (المنظم التربيعي الخطي (LQR)والمراقب الخطي الرباعي الخطي (LQG)مستندة على أساس مرشح كالمان) لتوهين وتضعيف الاضطراب وقمع أو تخميد الضوضاء.حيث تتزايد تطبيقات محرك التيار المستمر عديم الفرشاة ذو المغناطيس الدائم (PMBLDC) يومًا بعد يوم. عليه من أجل الحصول على الاستخدام المناسب لهذه المحركات والتحكم فيها بشكل فعال ، من المهم أن يكون لديك نمذجة رياضية مناسبة لهذه المحركات. وبالمثل ، فإن التحكم الفعال في هذه المحركات ضروري أيضًا للتطبيق الناجح للأجهزة عبر مجالات متعددة. تتناول هذه الورقة كلا الجانبين المهمين. تم اشتقاق نموذج رياضي لتمثيل نموذج محرك تيار مستمر بدون فرشاة ذو المغناطيس الدائم (PMBLDC) لدراسة الاستقرار والأداء. من أجل الحفاظ على الاستقرار وتحقيق أفضل أداء عن طريق تقليل توهين الاضطراب وقمع الضوضاء ، تم تطوير أدوات التحكم الثلاثة المثلى في هذا البحث. تم تقديم محاكاة أداء النظام لهذه وحدات التحكم المثلى مع وحدة التحكم الكلاسيكية PID باستخدام برنامج MATLAB للتحكم في محرك التيار المستمر بدون فرش (PMBLDC) ذي المغناطيس الدائم من أجل تخفيف الاضطراب وقمع الضوضاء .. وأظهرت نتائج المحاكاة ذلك و Linear Quadratic Gaussian (LQG) المستند على مرشح كالمان مع وحدة التحكم الكلاسيكية PID ، يوفر أفضل أداء مقارنة بوحدة التحكم PID ، والمنظم الخطي التربيعي (LQR) مع وحدة التحكم PID ومرشح كالمان مع وحدة التحكمPID .

Abstract

In this paper, modeling, simulation and performance analysis of the permanent magnet brushless direct current (PMBLDC) motor using classical controller (PID Controller) and optimal controllers (Linear Quadratic Regulator (LQR) and Linear Quadratic Gaussian (LQG) optimal Controllers Based on Kalman Filter) for disturbance attenuation and noise suppression is presented. The applications of the permanent magnet brushless direct current (PMBLDC) motor are increasing day by day. In order to have proper utilization of these motors and to control them effectively it is important to have proper mathematical modeling of these motors. Similarly effective control these motors are also essential to have successful application of the devices across multiple domains. This paper handles both these important aspects. A mathematical model has been derived to represent permanent magnet brushless direct current (PMBLDC) motor model to study the stability and performance. In order to maintain the stability and to achieve the best performance by reducing disturbance attenuation and noise suppression, the three optimal controllers are developed in this paper. the system performance simulation of these optimal controllers with PID controller is presented using MATLAB program to control the modeled permanent magnet brushless direct current (PMBLDC) motor for disturbance attenuation and noise suppression.. The simulation results show that and Linear Quadratic Gaussian (LQG) Based on Kalman Filter with PID controller provides best as compared to PID controller, Linear Quadratic Regulator (LQR) with PID controller and Kalman Filter with PID controller.

Keyword: *Permanent Magnet Brushless Direct Current (PMBLDC) motor, PID Controller, Linear Quadratic Regulator (LQR), Linear Quadratic Gaussian (LQG), Integral Linear Quadratic Regulator , MATLAB/Simulink etc.*

1. Introduction

Conventional DC motors have many attractive properties such as high efficiency and linear torque-speed characteristics. The control of DC motors is also simple and does not require complex hardware. However, the main drawback of the DC motor is the need of periodic maintenance. The brushes of the mechanical commutator eventually wear out and need to be replaced. The mechanical commutator has other undesirable effect such as sparks, acoustic noise and carbon particles coming from the brushes. Permanent magnet brushless direct current

(PMBLDC) motors can in many cases replace conventional DC motors. Despite the name, PMBLDC motors are actually a type of permanent magnet synchronous motors. They are driven by DC voltage but current commutation is done by solid state switches. The commutation instants are determined by the rotor position and the position of the rotor is detected either by position sensors or by sensorless techniques. PMBLDC motors have many advantages over conventional DC motors. A few of these are [1-2]: Long operating life, high dynamic response, high efficiency, better speed vs. torque characteristics, noiseless operation, higher speed range and higher torque-weight ratio. PMBLDC motors are available in many different power ratings, from very small motors as used in hard disk drives to large motors used in electric vehicles. Three-phase motors are most common but two-phase motors are also found in many application. The purpose of this paper is to build a simple, accurate and fast running Matlab model of a permanent magnet brushless direct current (PMBLDC) motor using Linear Quadratic Regulator (LQR) and Linear Quadratic Gaussian (LQG) optimal Controllers Based on Kalman Filter with PID controller for disturbance attenuation and noise suppression which will lead to an improvement in the transient and steady state response. This paper is organized as follows. Mathematical modeling of the three-phase permanent magnet brushless direct current (PMBLDC) motor is given in Sec. II. Optimal control strategies are given in Sec. III. Classical control strategy is given in Sec. IV. Analysis of Simulation Results is demonstrated in Sec. V. Conclusion is given in Sec. VI.

2. Mathematical Modeling Of The PMBLDC Motor

The mathematical model of the PMBLDC motor is fundamental for the corresponding performance analysis and control system design. The common mathematical models, which mainly include differential equation model, transfer function model, and state-space model, are presented as follow:

A. Differential Equation

The differential equation model is built for a three-phase two-pole PMBLDC motor [3]. Hence, the simplified schematic diagram of the motor can be obtained as shown in Fig. 1.

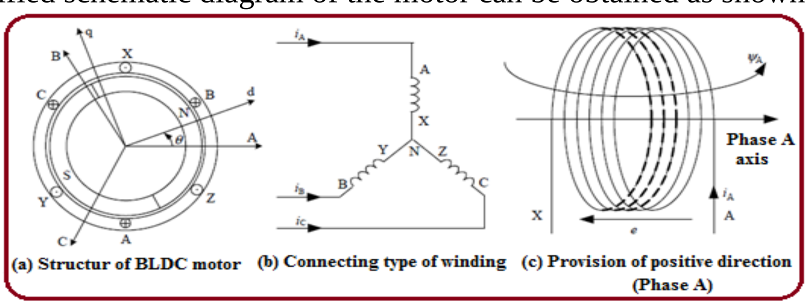

Figure.1 Schematic diagram of the PMBLDC motor.

Under the positive direction shown in Fig. 1, the phase voltage of each winding, which includes the resistance voltage drop and the induced EMF, can be expressed as

$$V_x = R_x i_x + e_{\psi x} \qquad (1)$$

Where

- V_x: → phase voltage, in which subscript x denotes phase A, B and C;
- i_x : → phase current.
- $e_{\psi x}$: → phase-induced EMF.
- R_x: → phase resistance. For three-phase symmetrical winding, there exists $R_A = R_B = R_C = R$).

The three-phase stator windings are symmetrical, the self inductances will be equal, and so as the mutual inductance. As for the three-phase symmetrical windings, there also exist fB(Θ) = fA(Θ -2π /3), and fC(Θ) = fA(Θ +2π /3). Then, the matrix form of phase voltage equation of PMBLDC motor can be expressed as

$$\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} + \begin{bmatrix} L-M & 0 & 0 \\ 0 & L-M & 0 \\ 0 & 0 & L-M \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} + \begin{bmatrix} e_A \\ e_B \\ e_C \end{bmatrix} \quad (2)$$

According to Equation (2), the equivalent circuit of the PMBLDC motor can be shown as in Fig.2.

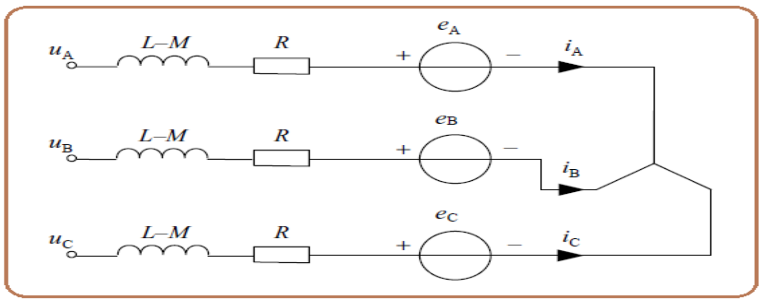

Figure.2 Equivalent circuit of the PMBLDC motor.

The power transferred to the rotor, which is called the electromagnetic power, the electromagnetic power is totally turned into kinetic energy equals the sum of the product of current and back-EMF of the three phases. That is

$$P_e = T_e\omega = e_A i_A + e_B i_B + e_C i_C \Rightarrow$$

$$T_e = \frac{e_A i_A + e_B i_B + e_C i_C}{\omega} \qquad (3)$$

Where

T_e : → electromagnetic torque;

ω: → angular velocity of rotation.

Substituting Equations, another form of the torque equation can be

$$T_e = P[\psi_m f_A(\theta)\, i_A + \psi_m f_B(\theta) i_B + \psi_m f_C(\theta) i_C] \quad (4)$$

Where

p : → is the number of pole pairs.

So Equation (4) can be further simplified as represented as

$$T_e = 2P\psi_m\, i_A = K_T i \qquad (5)$$

where

K_T: → the torque coefficient;

i : → the steady phase current.

In order to build a complete mathematical model of the electromechanical system, the motion equation has to be included as

$$T_e - T_L = J\frac{d\omega}{dt} + B_V\omega \qquad (6)$$

where

T_L : → load torque;

J : → rotor moment of inertia;

B_v : → viscous friction coefficient.

Thus, Equations (2), (3) and (6) constitute the differential equation mathematical model of the PMBLDC motor.

B. Transfer Functions

The three-phase PMBLDC motor is controlled by the full-bridge driving in the two-phase conduction mode, then when the windings of phase A and B are conducted, there exists

$$\begin{cases} i_A = -i_B = i \\ \frac{di_A}{dt} = -\frac{di_B}{dt} = \frac{di}{dt} \end{cases} \qquad (7)$$

Thus, the line-voltage V_{AB} can be rewritten as

$$V_{AB} = 2R\,i + 2(L-M)\frac{di}{dt} + (e_A - e_B) \qquad (8)$$

Take the transient process out of consideration (i.e. ignore the trapezoid bevel edge), then the steady e_A and e_B are equal in amplitude and opposite in direction when phases A and B are turned on. So, equation (8) can be expressed as

$$V_{AB} = V_d = 2R\,i + 2(L-M)\frac{di}{dt} + 2e_A$$

$$V_{AB} = V_d = r_a i + L_a\frac{di}{dt} + k_e\omega \qquad (9)$$

where

V_d : → ***DC*** bus voltage;

r_a : → line resistance of winding, $r_a = 2R$;

L_a : → equivalent line inductance of winding, $L_a = 2(L-M)$;

k_e : → coefficient of line back-EMF, $k_e = 2p\psi_m = 4pNSB_m$.

The transfer function of a PMBLDC motor with no load can be expressed as

$$G_u(s) = \frac{\omega(s)}{V_d(s)} = \frac{K_T}{L_aJs^2 + (r_aJ + L_aB_V)s + (r_aB_V + K_Tk_e)} \qquad (10)$$

In the following, the transfer function of a PMBLDC motor when the load torque is not zero, it is shown in Fig. 3.

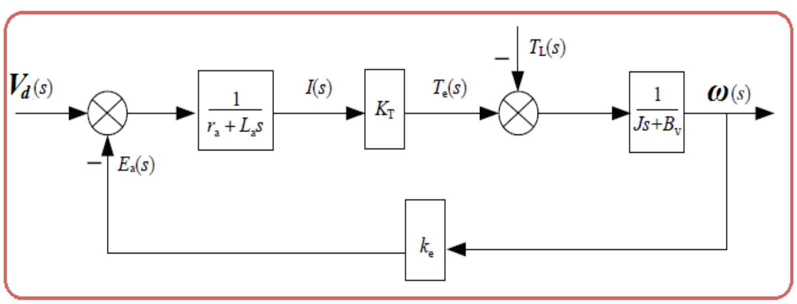

Figure.3 Structure diagram of PMBLDC motor with load torque.

For such a system, the superposition principle holds. Thus, the output of the system equals the sum of outputs when $V_d(s)$ and $T_L(s)$ are applied to the system, respectively. In Fig. 3, when $V_d(s)=0$ holds, then, the transfer function between load torque and speed is

$$G_L(s) = \frac{\omega(s)}{T_L(s)} = -\frac{r_a + L_as}{L_aJs^2 + (r_aJ + L_aB_V)s + (r_aB_V + K_Tk_e)} \qquad (11)$$

Therefore, the speed response of a PMBLDC motor affected together by voltage and load torque is given by

$$\omega(s) = G_u(s)V_d(s) + G_L(s)T_L(s)$$

$$\omega(s) = \frac{K_T V_d(s) - (r_a + L_a s)T_L(s)}{L_a J s^2 + (r_a J + L_a B_V)s + (r_a B_V + K_T k_e)} \quad (12)$$

C. State-Space Equations

The state-space equation method is one of the most important analysis methods in modern control theory. From the state equation we can get all the independent variables and then determine all the motion states of the system. A group of first-order differential equations with state variables is used in the state-space method to describe the dynamic characteristics of the system. Since it is helpful to the realization of different digital control algorithms, the state-space method is becoming more and more popular in designing control systems with the fast development of computer techniques. Especially in recent years, computer on-line control systems such as optimal control, Kalman filters, dynamic system identification, self-adaptive filters and self adaptive control have been applied to motor control. All these control techniques are based on the state equation. Currents of three phase windings and the angular speed are selected here as state variables, and the fourth-order state equation is then derived as

$$\dot{X} = Ax + Bu \qquad (13)$$

Where

$$X = [i_A \quad i_B \quad i_C \quad \omega\ ]^T$$
$$V = [V_A \quad V_B \quad V_C \quad T_L\ ]^T$$

$$A = \begin{bmatrix} \frac{-R}{L-M} & 0 & 0 & \frac{-P\psi_{pm}(\theta)}{L-M} \\ 0 & \frac{-R}{L-M} & 0 & \frac{-P\psi_{pm}\left(\theta - \frac{2\pi}{3}\right)}{L-M} \\ 0 & 0 & \frac{-R}{L-M} & \frac{-P\psi_{pm}\left(\theta - \frac{4\pi}{3}\right)}{L-M} \\ \frac{P\psi_{pm}(\theta)}{J} & \frac{P\psi_{pm}\left(\theta - \frac{2\pi}{3}\right)}{J} & \frac{P\psi_{pm}\left(\theta - \frac{4\pi}{3}\right)}{J} & \frac{-B_V}{J} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{1}{L-M} & 0 & 0 & 0 \\ 0 & \frac{1}{L-M} & 0 & 0 \\ 0 & 0 & \frac{1}{L-M} & 0 \\ 0 & 0 & 0 & \frac{-1}{J} \end{bmatrix}$$

The controllability of a linear system is the base of optimal control and optimal estimation, so it should be determined. Assume the controllability matrix is

$$M = [M_0 \quad M_1 \quad M_2 \quad M_3\ ] \qquad (14)$$

Where

$$M_0 = B, \qquad M_i(t) = A^i B, \qquad i = 1, 2, 3.$$

Then, matrix M can be transformed to

$$M = \begin{bmatrix} \lambda & 0 & 0 & 0 & & & \\ 0 & \lambda & 0 & 0 & & & \\ 0 & 0 & \lambda & 0 & M_1 & M_2 & M_3 \\ 0 & 0 & 0 & -\frac{1}{J} & & & \end{bmatrix} \qquad (15)$$

Where

$$\boldsymbol{\lambda} = \mathbf{1}/(\boldsymbol{L} - \boldsymbol{M})$$

The matrix $\boldsymbol{M}$ meets the condition of rank [$\boldsymbol{M}$] = 4. So, the system represented by equation (13) is controllable and all the poles of the system can be arbitrarily placed by state feedback.

3. Optimal Control Strategy

A. Linear Quadratic Regulator (LQR) Control System

The linear quadratic regulator (LQR) is an optimal controller that provides practical state feedback gain matrix. The controller has been used for minimizing the cost function [4]. The LQR is a state feedback control technique that computes optimal feedback gain matrices for given states pace represented systems with respect to a quadratic cost function which is minimized [5]. The feedback gain matrix is associated to a solution of the Riccati equation. The LQR provides an optimal control law with quadratic performance index or quadratic cost function where the system dynamics are described as a set of differential equations [6]. The LQR approach deals with the optimization of a cost function or performance index. Thus, the designer can weigh which states and which inputs are more important in the control action to seek for appropriate transient and steady-state performances [7-8]. The optimal control problem is to find a control u which causes the system

$$\dot{\boldsymbol{X}} = \boldsymbol{g}(\boldsymbol{x}(\boldsymbol{t}), \boldsymbol{u}(\boldsymbol{t}), \boldsymbol{t}) \qquad (\mathbf{16})$$

To follow an optimal trajectory x(t) that minimizes the performance criterion, or cost function

$$\boldsymbol{J} = \int_{t_0}^{t_1} \boldsymbol{h}(\boldsymbol{x}(\boldsymbol{t}), \boldsymbol{u}(\boldsymbol{t}), \boldsymbol{t})\boldsymbol{dt} \quad (\mathbf{17})$$

The problem is one of contrained functional minimization a quadratic performance index or quadratic cost function is

$$\boldsymbol{J} = \int_{0}^{\infty} (\boldsymbol{x}^{T}\boldsymbol{Q}\boldsymbol{x} + \boldsymbol{u}^{T}\boldsymbol{R}\boldsymbol{u})\boldsymbol{dt} \quad (\mathbf{18})$$

Where

Q: → State weighting matrix (square, symmetric and non-negative definite)

R: → Control weighting matrix (square, symmetric and positive definite)

J: → Is a scalar quantity

The optimal control law or state feedback law is

$$\boldsymbol{U}(\boldsymbol{t}) = -\boldsymbol{K}\boldsymbol{x}(\boldsymbol{t}) \qquad (\mathbf{19})$$

Where

K: → Is the controller gain or state feedback gain matrix and a value of K that will produce a desired set of
closed-loop poles

The state feedback gain matrix (K) is found by [8]

$$\boldsymbol{K} = \boldsymbol{R}^{-1}\boldsymbol{B}^{T}\boldsymbol{P} \qquad (\mathbf{20})$$

Where

B: → Is the input matrix of the plant (BLDC motor).

P: → Is the unique positive definite solution to algebraic Riccati equation or matrix Reccati equation.

The feedback gain matrix (K) is associated to a solution of the Riccati equation (P). Therefore, the continuous solution of the matrix Riccati equation or algebraic Riccati equation is

$$Q + PA + A^T P - PBR^{-1} B^T P = 0 \qquad (21)$$

Where

A: → Is the system matrix of the plant

By knowing the state matrices A and B and properly selecting Q and R, the value of K can be obtained. The function in MATLAB can be used Lqr(sys,Q,R,N). The discrete quadratic performance index or discrete quadratic cost function is

$$J = \sum_{K=0}^{N-1} \left(x^T(k) Q x(k) + u^T(k) R u(k) \right) T \quad (22)$$

The discrete solution of the state equation is

$$X(k+1) = A(T)x(k) + B(T)u(k) \qquad (23)$$

Where

T: → Is the sampling time of a discrete-time system

A(T) : → Is the discrete-time state transition matrix $A(T) = e^{A\tau}$

B(T) : →Is the discrete-time control matrix $B(T) = \int_0^T e^{A\tau} B d\tau$

The discrete solution of the matrix Riccati equation solves recursively for K and P in reverse time, commencing at the terminal time, where [6]

$$K(N-(k+1)) = [TR + B^T(T)P(N-k)B(T)]^{-1} B^T(T)P(N-k)A(T) \quad (24)$$

And

$$P(N-(k+1)) = \left[TQ + K^T\left(N-(k+1)\right)TRK(N-(k+1))\right] + \left[A(T) - B(T)K\left(N-(k+1)\right)\right]^T P(N-k)\left[A(T) - B(T)K\left(N-(k+)\right)\right] \quad (25)$$

As K is increased from 0 to N-1, the algorithm proceeds in reverse time, when run in forward-time, the optimal control law at step k is

$$U(k) = -K(k)x(k) \qquad (26)$$

The LQR design technique has certain advantages over the classical control design methods or Eigen-structure assignment based methods, as it guarantees adequate stability margins [5]. On the other hand, there are some limitations the given system must satisfy e.g. it must be stabilizable and free of non-observable states which means The LQR approach requires the knowledge of all state variables [9]. The performance of LQR may be deviated due to the presence of system noise [4]. The LQR in its basic form forces the controlled states to reach zero, which is a known regulation problem. In order to transform the regulation capability to command tracking, an integral error dynamics must be considered to remove the steady state error. Fig.4 shows Linear quadratic regulator LQR control system

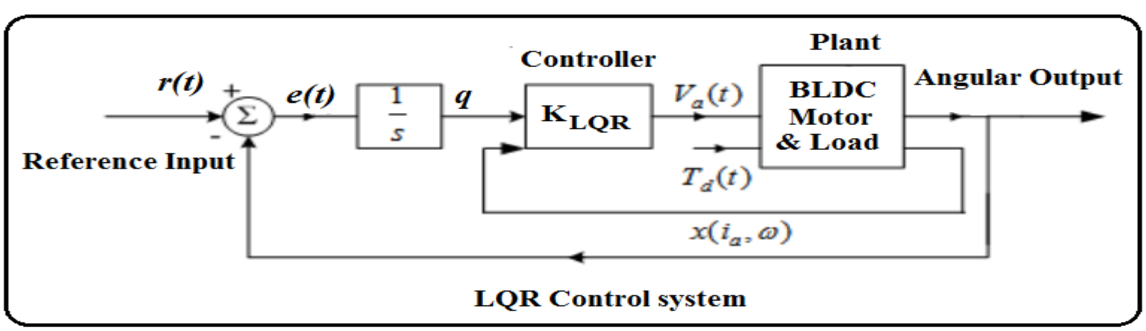

Figure.4 LQR Control System

A. Kalman Filter State Estimator

Kalman Filter is used for both filtering and state estimation purpose. In the design of state observers, it was assumed that the measurements y=Cx were noise free. In practice, this is not usually the case and therefore the observed state vector $\hat{x}$ may also be contaminated with noise. The state estimation is the process of extracting a best estimate of a variable from a number of measurements that contains noise. The classical problem of obtaining a best estimate of a signal by combining two noisy continuous measurements of the same signal was first solved by Weiner (1949), his solution required that both the signal and noise be modeled as random process with known statistical properties. This work was extended process based upon an optimal minimum variance filter, generally referred to as a kalman filter. The kalman filter is a complementary form of the Weiner filter. The plant is subject to a Gaussian sequence of disturbance *w(kT)* with disturbance transition matrix Cd(T). Measurements z(k+1)T contain a Gaussian noise sequence v(k+1)T as shown in fig.5 [6]

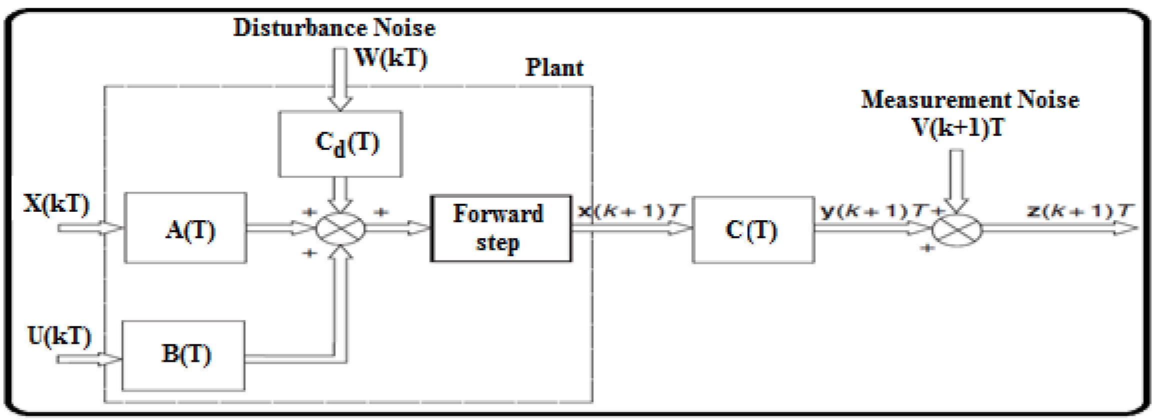

Figure.5 Plant with disturbance and measurements noise

The general form of the Kalman filter usually contains a discrete model of the system together with a set of recursive equations that continuously update the Kalman gain matrix [K] and the system covariance matrix [P]. the optimal value of the Kalman gain matrix [K] is the one that yields the minimtate um variance [P]. The state estimate $\hat{x}(k+1/k+1)$ is obtained calculating the predicted state $\hat{x}(k+1/k)$ from

$$\hat{\boldsymbol{x}}(\boldsymbol{k}+\mathbf{1}/\boldsymbol{k})\boldsymbol{T} = \boldsymbol{A}(\boldsymbol{T})\hat{\boldsymbol{x}}(\boldsymbol{k}/\boldsymbol{k})\boldsymbol{T} + \boldsymbol{B}(\boldsymbol{T})\mathbf{U(T)} \qquad \textbf{(27)}$$

And the determine the estimated state at time (k+1)T using

$$\hat{\boldsymbol{x}}(\boldsymbol{k}+\mathbf{1}/\boldsymbol{k}+\mathbf{1})\boldsymbol{T} = \hat{\boldsymbol{x}}(\boldsymbol{k}+\mathbf{1}/\boldsymbol{k})\boldsymbol{T} + \boldsymbol{K}(\boldsymbol{k}+\mathbf{1})[\boldsymbol{Z}(\boldsymbol{k}+\mathbf{1})\boldsymbol{T} - \boldsymbol{C}(\boldsymbol{T})\hat{\boldsymbol{x}}(\boldsymbol{k}+\mathbf{1}/\boldsymbol{k})\boldsymbol{T}] \qquad \textbf{(28)}$$

Where

The term (k/k) : → means data at time k based on information available at time k.
The term (k+1/k) : → means data at time (k+1) based on information available at time k.
The term (k+1/k+1) : → means data at time (k+1) based on information available at time (k+1).

The vector of measurements is given by

$$\boldsymbol{Z}(\boldsymbol{k}+\mathbf{1})\boldsymbol{T} = \boldsymbol{C}(\boldsymbol{T})\boldsymbol{x}\boldsymbol{k}+\mathbf{1})\boldsymbol{T} + \boldsymbol{V}(\boldsymbol{k}+\mathbf{1})\boldsymbol{T} \qquad \textbf{(29)}$$

Where

$Z(k+1)T$: → is the measurement vector
$C(T)$: → is the measurement matrix
$V(k+1)T$: → is a Gaussian noise sequence

The kalman gain matrix [K] is obtained from a set of recursive equations that commence from some initial covariance matrix (P(k/k)).

$$P(k+1/k) = A(T)P(k/k)A^T(T) + C_d(T)QC_d^T(T) \qquad (30)$$

$$K(k+1) = P(k+1/k)C^T(T)[C(T)P(k+1/K)C^T(T) + R]^{-1} \qquad (31)$$

$$P(k+1/k+1) = [I - K(k+1)C(T)]P(k+1/k) \qquad (32)$$

Where

$C_d(T)$: → is the disturbance transition matrix
Q: → is the disturbance noise covariance matrix
R: → is the measurement noise covariance matrix

The recursive process continues by substituting the covariance matrix P(k+1/k+1) computed in equation (32) back into equation (30) as P(k/k) until K(k+1) settles to a steady value. Equations (23) to (32) are illustrated in fig.6 [6] which shows the block diagram of the Kalman filter is

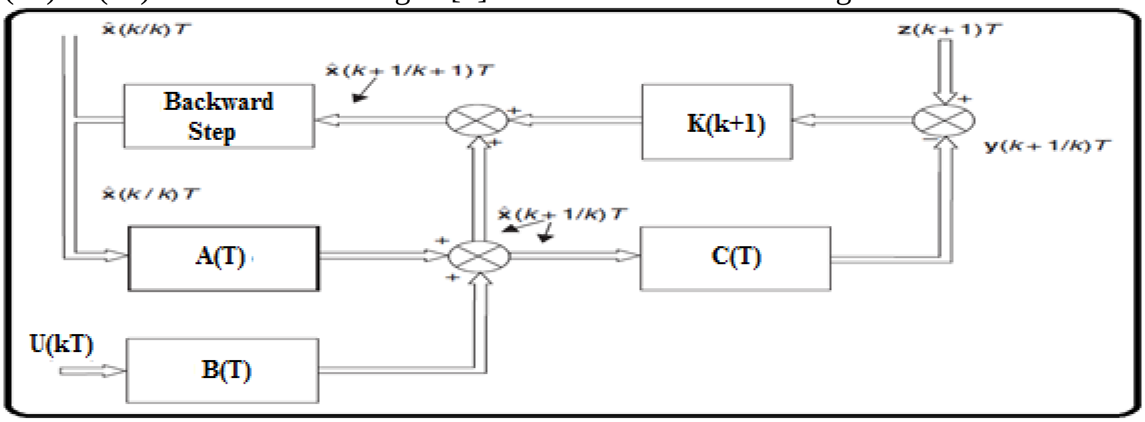

Figure.6 The Kalman Filter

The difference is that the Kalman filter is computed in forward-time, the LQR being computed in reverse-time

C. Linear Quadratic Gaussian (LQG) control system

A control system that contain a LQRegulator/Tracking controller together with a Kalman filter state estimator is called a Linear Quadratic Gaussian (LQG) control system. LQG is shown in fig. 7.

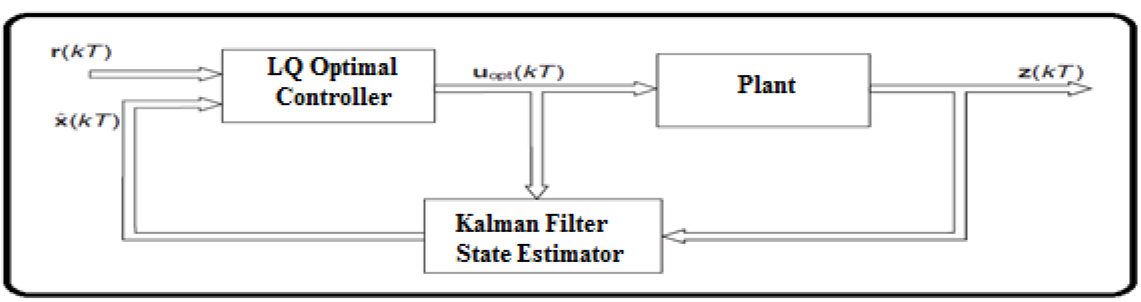

Figure.7 Linear Quadratic Gaussian (LQG) control system

LQG control can be applied to linear time invariant systems as well as linear time variant systems. It deals with uncertain linear systems disturbed by additive white gaussian noise, having incomplete state information. Practically it is used for predicting future courses of dynamic systems. Designing of optimal LQG controller with the feedback controller designed in such a way that it minimizes cost function [4].

$$J = \lim_{n\to\infty} E[\frac{1}{T}\int_0^T (x^T(t)Qx(t) + u^T(t)Ru(t))dt] \quad (33)$$

Where

$Q \geq 0$ *and* $R > 0$: →are symmetric weighting matrices.

$E[\cdot]$: →is the expected value

In the cost function, the term x^TQx corresponds to a requirement to minimize the states of the system. the term u^TRu corresponds to the requirement to minimize the size of control inputs. The selection of matrices Q and R in the cost in the cost function depends on the desired performance objective of the system. Minimizing the tracking error between the command signal and measured output is the main control objective. The continuous time solution to the optimal observer problem is [8]

$$L = P_0C^TR_0^{-1} \quad (34)$$

Where $P_0$is the solution of the algebraic Riccati equation:

$$AP_0 + P_0A^T - P_0C^TR_0^{-1}CP_0 + Q_0 = 0 \quad (35)$$

The calculation was executed in Matlab using the kalman(sys,Qn,Rn,Nn) function. The function returns the discrete observer gain vector L if the system sys is in discrete time. The block diagram of the complete LQG controller can be seen on fig. 8 [10].

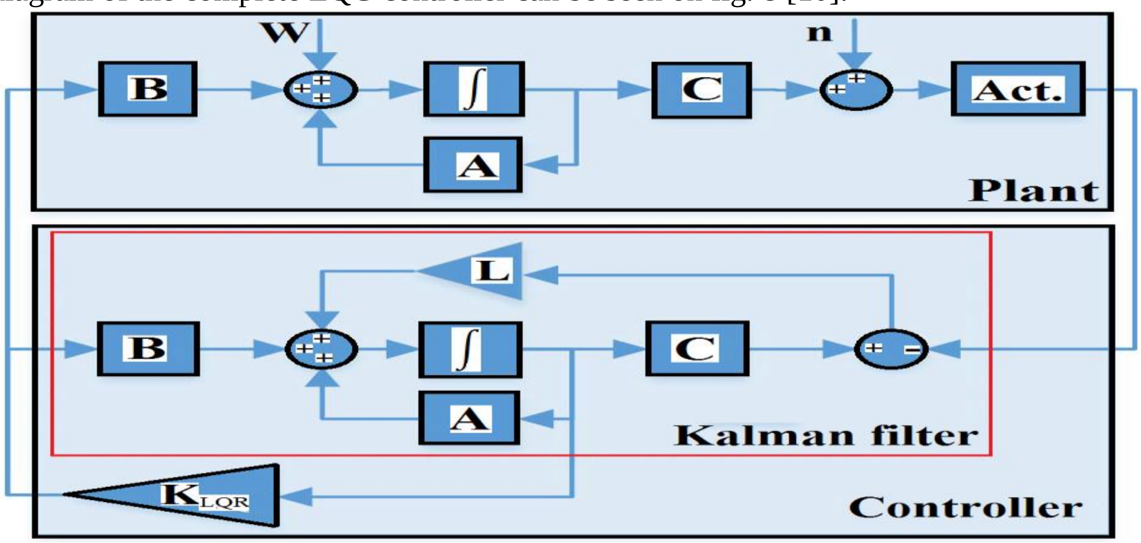

Figure.8 The LQG control system scheme.

Fig.8 shows the combination of the feedback gain matrix K_{LQR} and a Kalman Filter in closed loop with a state-space Plant description. A major advantage of the LQG controller design approach lies in the possibility to estimate the missing states, i.e. so the designer doesn't need to have a complete knowledge of the state vector. The next advantage is in the noise attenuation capabilities. However the cost for these features is in sacrificing the system's closed-loop robustness.

4. CLASSICAL CONTROL STRATEGY

A. PID CONTROLLER

Fundamentally, PID controllers are composed of three basic control actions. They are simple to implement and provide better performance. The tuning process of the gains of PID controllers can be complex because it is iterative. First, it is necessary to tune the "Proportional" mode, then the "Integral", and then add the "Derivative" mode to stabilize the overshoot, then add more "Proportional", and so on. The PID controller has the following form in the time domain

$$U(t) = K_p e(t) + K_i \int_0^t e(t)dt + K_d \frac{de(t)}{dt} \qquad (36)$$

Where

e(t): → is the system error (difference between the reference input and the system output).
u(t) : → is the control variable.
Kp: → is the proportional gain.
Ki : → is the integral gain.
Kd : → is the derivative gain.

The effects of these parameters on the output response of the system are shown in Table 1 [11]. A PID controller does not "know" the correct output to bring the system to the set point. It moves the output in the direction which should move the process toward the set point and needs to have feedback (measurements) to perform. Using the Laplace Transform for equation (36) and assuming initial conditions equal to zero the transfer function of the PID can be written as

$$G(s) = \frac{U(s)}{E(s)} = K_p + \frac{K_i}{s} + K_d s \qquad (37)$$

Transfer function of a PID controller is rearranged, the three terms can be recognized follows:

$$G(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right) = K_p\left(\frac{T_i T_d s^2 + T_i s + 1}{T_i s}\right) \qquad (38)$$

Where:

Ti=Kp/Ki : → is the integral time constant
Td=Kd/Kp : → is the derivative time constant

Table 1: Effect of PID parameter on system response

Parameter	Rise-Time	Overshoot	Settling Time	Sready State Error	Stability
K_p	Decrease	Increase	Small Change	Decrease	Decrease
K_i	Decrease	Increase	Increase	Eliminate	Decrease
K_d	Minor Changes	Decrease	Decrease	No Effect	Improve if Kd is small

The selection of the Proportional Integral and Derivative (PID) controller parameters can be obtained using the Ziegler-Nichols method , trail and error method or other tuning methods. In terms of Ziegler-Nichols method, the PID controller parameters can be found depending on the values of as shown in fig.9 and using the Ziegler-Nichols equations.

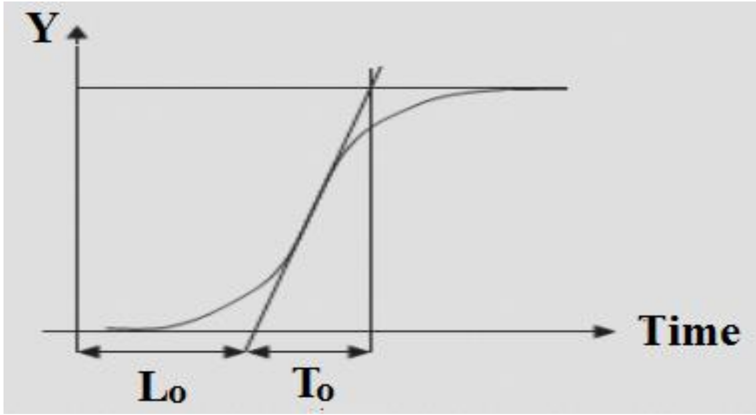

Figure.9 Zegiler-Nichols PID Parameters

Now, using the following equations, the PID parameters can be derived:

$$K_p = 1.2\frac{T_o}{L_O} \quad (39)$$

$$T_i = 2L_O \quad (40)$$

$$T_d = 0.5L_O \quad (41)$$

In this paper, the PID controller parameters can be obtained by the help of optimal control methods

5. Analysis Of Simulation Results Of LQR And LQG Optimal Controllers Based On Kalman Filter

The modeling of three phase permanent magnet brushless direct current (PMBLDC) motor with classical and optimal controllers has been derived. in addition to that, simulation and performance analysis of the PMBLDC motor with and without optimal controllers have been implemented and investigated by using MATLAB/SIMULINK software. The goal of control engineering design is to obtain the configuration, specifications, and identification of the key parameters of a proposed system to meet an actual need [12-13]. Establishment of goals and variables to be controlled, The most basic requirement of PMBLDC motor is that it should rotate at the desired output response (desired value or reference input), as well as, optimals controllers are used for reducing the sensitivity of the actual output response to external load (external disturbances), load variations (changes in the torque opposed by the motor load), noise and parameters changes , where the actual output response variations induced by such disturbances must be minimized .

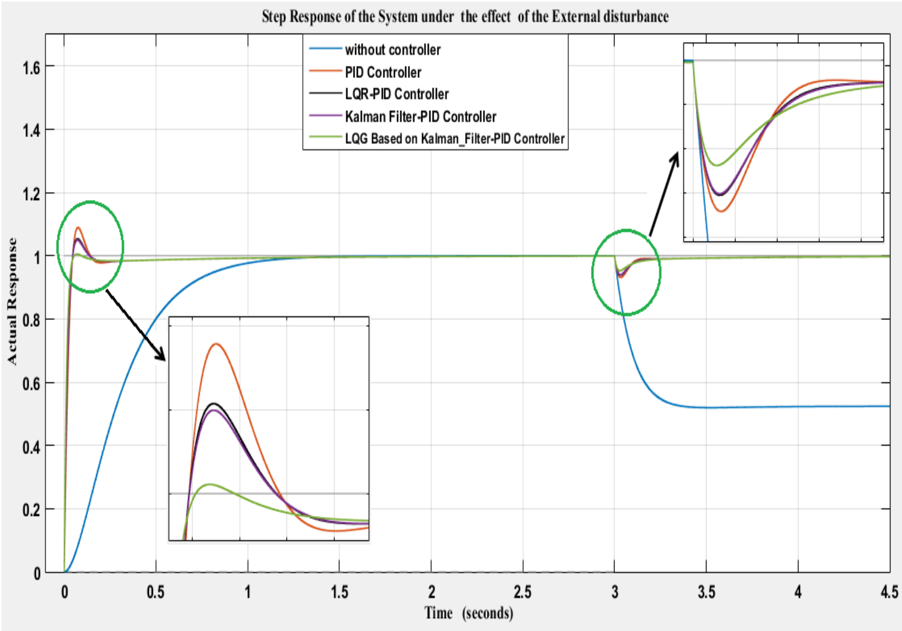

Figure.10 Simulation results of the PMBLDC motor with PID, LQR-PID, Kalman Filter-PID and LQG based on Kalman Filter-PID controllers.

Table 2: A comparison of the simulation results of the PMBLDC motor with classical and optimal control strategy in terms of time response specifications

The PMBLDC Motor with the Effect of External Disturbance				
Time Domain Specifications	Strategy of Control			
	Classical Control Strategy	Optimal Control Strategy		
	PID Controller	LQR-PID Controller	Kalman Filter-PID Controller	LQG Based on Kalman_Filter-PID Controller
Settling Time (t_s)	0.2420 Sec	0.1134sec	0.1117 sec	0.0421 sec
Maximum Overshoot (M_p)	8.9353 %	5.3674%	4.9728 %	0.54333%
Peak Time (t_p)	0.075 Sec	0.072sec	0.072 sec	0.068 sec
Rise Time (t_r)	0.034 Sec	0.031sec	0.0315 sec	0.026 sec
Delay Time (t_d)	0.0133 sec	0.0103 sec	0.01 sec	0.0061 sec
Steady state error (e_{ss})	0.000030477	0.000028722	0.000027152	0.000024772
Damping ratio (ζ)	0.60948	0.68141	0.69076	0.85659

Table 3: A comparison of the simulation results of the PMBLDC motor with classical and optimal control strategy in terms of frequency response specifications

The PMBLDC Motor with the Effect of External Disturbance				
Frequency Domain Specifications	Strategy of Control			
	Classical Control Strategy	Optimal Control Strategy		
	PID Controller	LQR-PID Controller	Kalman Filter-PID Controller	LQG Based on Kalman_Filter-PID Controller
Phase Margin (P.M)	84.5691 ^{0}	88.115 ^{0}	88.4233 ^{0}	90.9835 ^{0}
Gain Margin (G.M)	Inf dB	Inf dB	Inf dB	Inf dB
Bandwidth (ω_b)	121.6004 Hz	148.7355 H	151.1262 Hz	208.8586 Hz
Resonant Peak (M_r)	1.0348	1.0026	1.0010	1.1312
Resonant Frequency (ω_r)	54.2915 Hz	38.3434 Hz	31.5745 Hz	Damping ratio (ζ) =0.85

Table 4: Comparison for all performance indices parameters of the PMBLDC motor with classical and optimal control strategy

The Three Phase PMBLDC Motor without the effect of external load [$T_L(s)$]				
Strategy of Control	Performance Criteria			
	IAE $IAE = \int_0^t \lvert e(t)\rvert dt$	ITAE $ITAE = \int_0^t t\lvert e(t)\rvert dt$	ISE $ISE = \int_0^t (e(t))^2 dt$	ITSE $ITSE = \int_0^t t(e(t))^2 dt$
PID Controller	0.0001523856	0.0003809641	0.0000000046	0.0000000116
LQR-PID Controller	0.0001436105	0.0003590263	0.0000000041	0.0000000103
Kalman Filter-PID Controller	0.0001357600	0.0003394001	0.0000000037	0.0000000092
LQG Based on Kalman_Filter-PID Controller	0.0001238615	0.0003096539	0.0000000031	0.0000000077

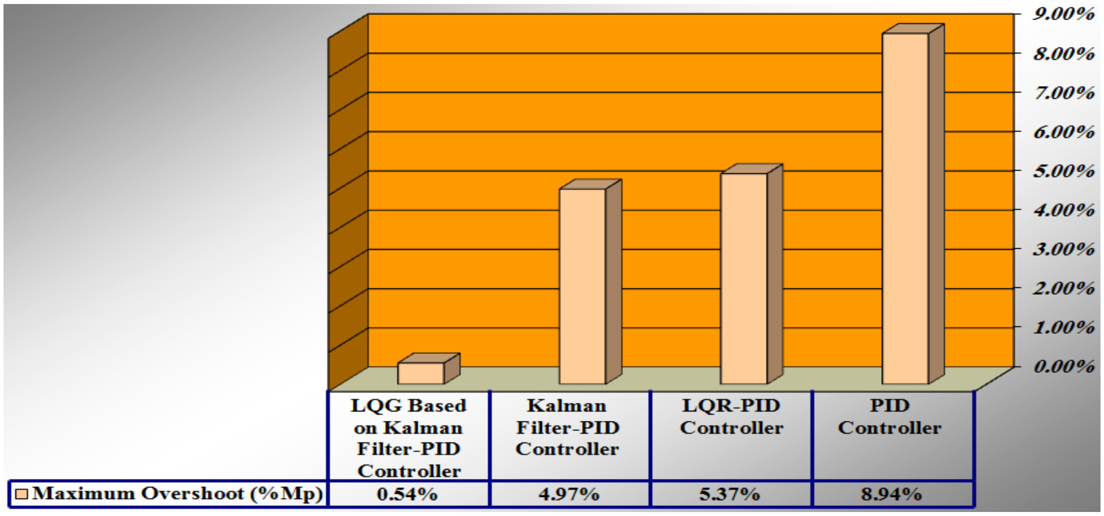

Figure.11 Comparison of maximum overshoot (%Mp) for classical and optimal control strategy

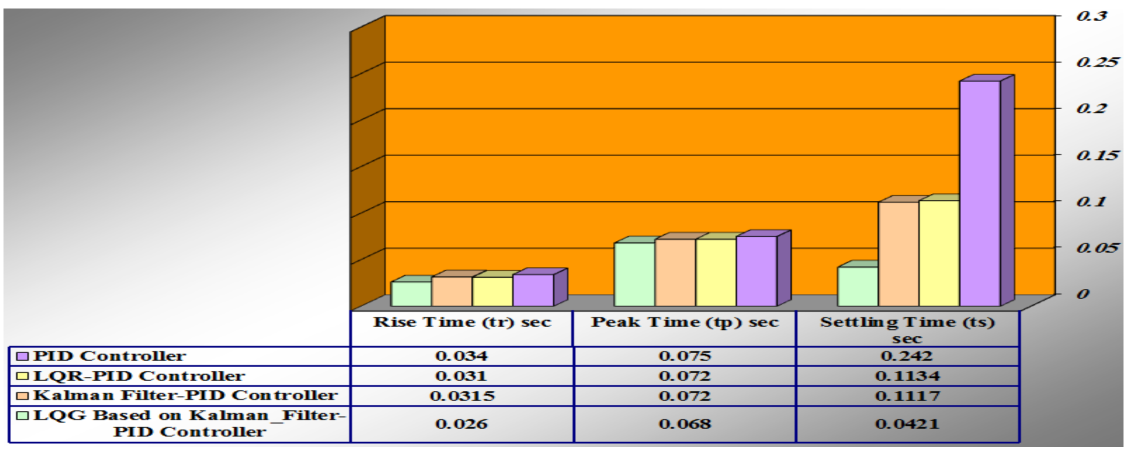

Figure.12 Comparison of Rise Time (t_r), Peak Time (t_p) and Setting Time (t_s) for classical and optimal control strategy

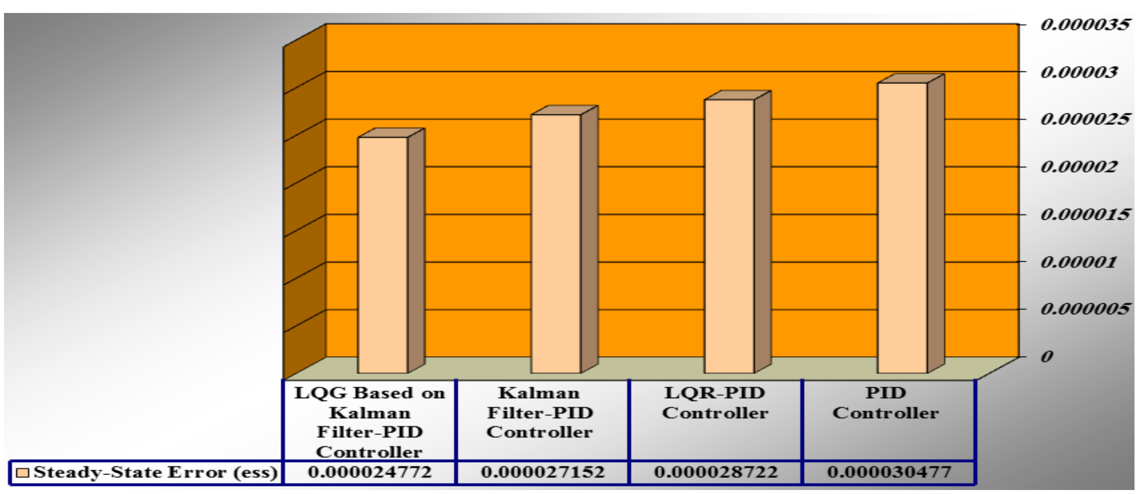

Figure.13 Comparison of Steady State Error (e_{ss}) for classical and optimal control strategy

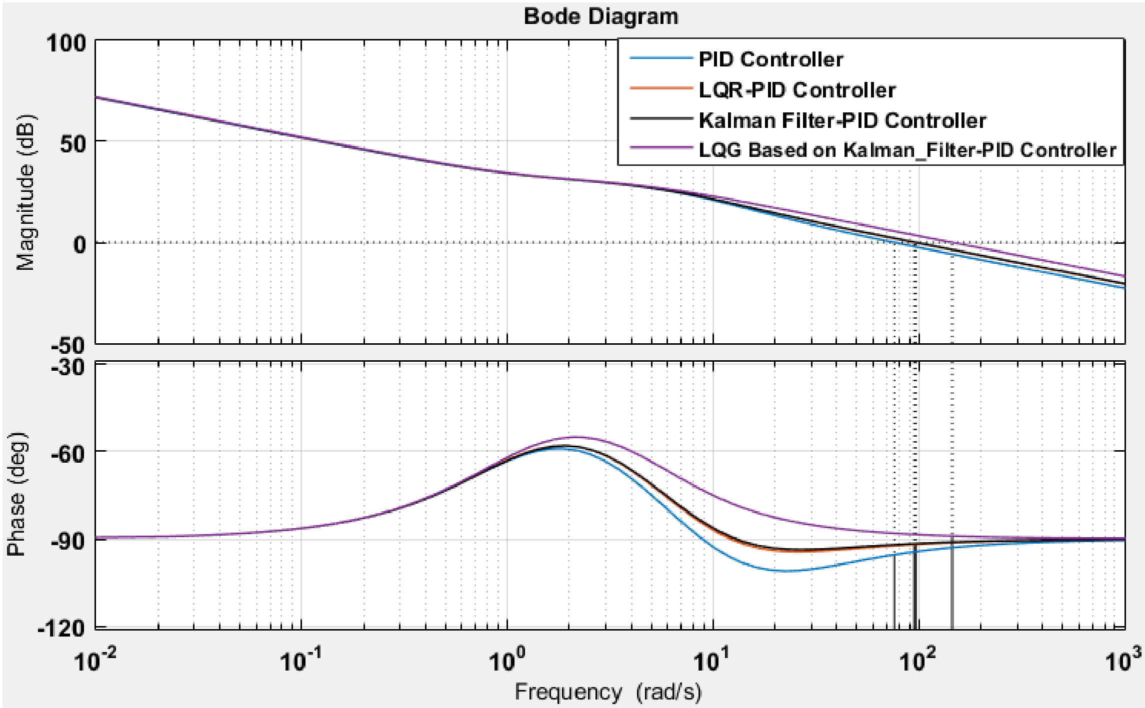

Figure.14 Bode Diagram of the PMBLDC motor with PID, LQR-PID, Kalman Filter-PID and LQR based on Kalman Filter-PID controllers

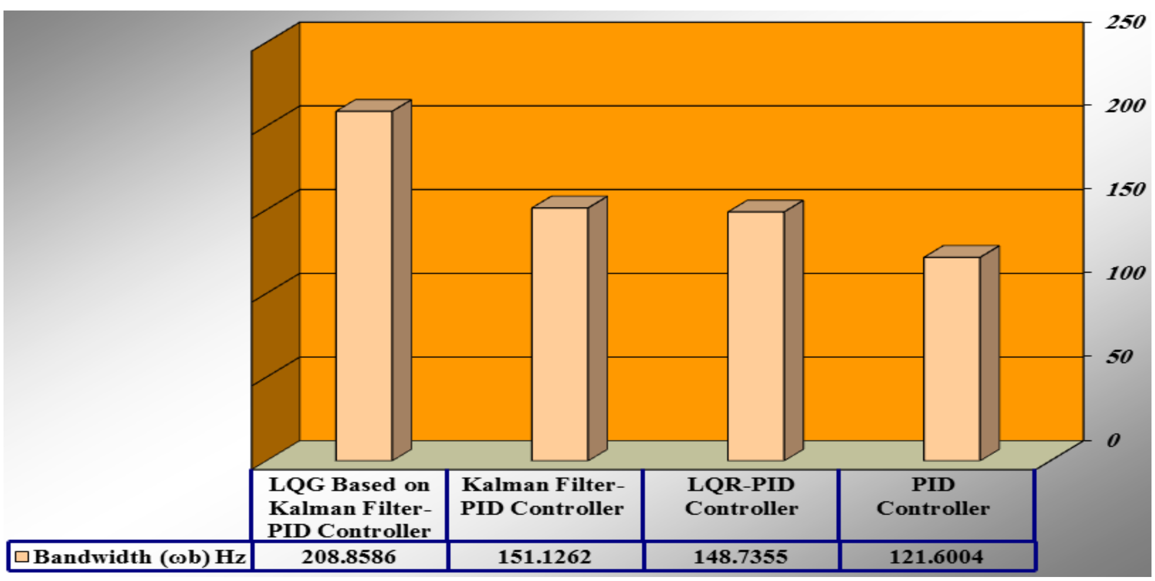

Figure.15 Comparison of Bandwidth (ω_b) for classica
l and optimal control strategy

As a result of the simulation, LQG based on Kalman Filter-PID controller is the best controller compared to other controllers, which presented satisfactory performances, process good robustness and also perfect speed tracking. The main objective of controllers is to minimize the error signal or in other words the minimization of performance criteria. Therefore, A set of performance indicators (Integral Absolute Error (IAE), Integral Time Absolute Error (ITAE), Integral Square Error (ISE), Integral Time-weighted Squared Error (ITSE)) have been used as a design tool aimed to evaluate tuning methods results. Performance criteria shows the superiority of , LQG based on Kalman Filter-PID control method over PID, LQR-PID and Kalman Filter-PID control methods.

6 . Conclusion

This paper has demonstrated that the performance of a BLDC motor can be improved by using LQR and LQG optimal Controllers Based on Kalman Filter with PID controller for disturbance attenuation and noise suppression. The actual output response of the permanent magnet brushless direct current (PMBLDC) Motor is controlled by means of the PID control method, LQR-PID control method, and LQG based on Kalman filter PID control method for enhancement the stability and accuracy under the effect of load variations, external disturbances, noise and parameters changes. In this paper, with reference to the results of the computer simulation by using (MATLAB & SIMULINK) software, the performance characteristics of classical and optimal controllers are compared in terms of the time response and frequency response. The simulation results illustrate that LQG Based on Kalman Filter-PID Control method performs better than PID, LQR-PID and Kalman filter-PID control method, and has verified all design requirements of the system. LQG Based on Kalman Filter-PID Control method is the best Controller which presented satisfactory performances and possesses good robustness This control method seems to have a lot of promise in the real world application.

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