

Annular Friction and Cuttings Density Effects on Pressure Margins in Belhedan Fractured Carbonate Wells

Emad Aljabali*, Naser A. Altoumi, Eman A. Khmakem

Department of Petroleum Engineering, Faculty of Oil, Gas and Renewable Energy Engineering,
University of Zawia, Zawia, Libya

*e.aljabali@zu.edu.ly

Abstract

Annular friction and suspended cuttings can consume much of the pressure allowance in fractured carbonate wells even when annular losses are below 4% of standpipe pressure. This study uses actual field operating inputs and WellPlan hydraulic outputs from Belhedan wells V33, V36, and V39 to compare segmented Bingham-flow models and evaluate solids-loading effects against local pressure reserves. Exact concentric-annulus and plane-slot solutions separate curvature error from plug-flow truncation. At fixed flow rate, the relative pressure-gradient error of the linearized slot is $\xi^3/2$ (requiring plug ratio $\xi \leq 0.464159$ for a 5% tolerance), whereas a 1,860-state dimensionless survey ($0.20 \leq \kappa \leq 0.80$, $1 \leq Bi \leq 60$) finds a maximum slot-to-annulus curvature error of 3.36%. Across the wells, the exact solutions differ by 0.10–0.12 psi, whereas linearization adds 6.97–9.24 psi. At 1 vol% solids, the calculated margin sign differs across models in V39 (+5.78 vs. –3.59 psi, consuming 162% of the concentric reserve; +5.44 psi with micromechanical coupling) at diagnostic specific gravity 2.145, and in V33 (+1.28 vs. –1.75 and –5.79 psi) and V36 (+0.69 vs. –2.83 and –7.37 psi) at calcite reference density 2.71. Model-specific critical formation limits ($F_{crit,m}$) delimit where hydraulic model choice changes the margin classification.

Keywords: Bingham annular flow; plug ratio; approximation error; suspension rheology; pressure margin; fractured carbonates.

الملخص

تزيد فواقد الاحتكاك في الفراغ الحلقي وفترات الصخور العالق الضغط المؤثر على التكوين أثناء الدوران، مما قد يقلل هامش الضغط المتاح، خاصة في التكوينات الكربوناتيّة المتشققة. تهدف هذه الدراسة إلى تقييم تأثير طرق حساب الاحتكاك وتركيز الفتات وكثافته في هامش الضغط، بالاعتماد على بيانات تشغيلية حقيقية فعلية ومخرجات برنامج WellPlan للأبار V33 و V36 و V39 في حقل بلحيدان. استُخدم نموذج بينغهام لمقارنة الحل الدقيق للجريان في الفراغ الحلقي المترافق والشق المستوي المكافئ بالتقريب الخطي المبسط، إلى جانب اختبار 1,860 حالة حسابية وتقييم أثر الفتات في كثافة السائل ولزوجته وإجهاد خضوعه. أظهرت النتائج تقارباً وثيقاً بين الحلين الحلقي والشق المستوي بفارق 0.10–0.12 رطل/بوصة²، بينما زاد التقريب الخطي فواقد الضغط المحسوبة بمقدار 6.97–9.24 رطل/بوصة². وعند إضافة فتات صخري بنسبة حجمية 1% وكثافة نوعية 2.145، بقي هامش الضغط في البئر V39 موجباً وفق الحل الدقيق، وأصبح سالباً وفق التقريب الخطي عند الحد المرجعي المعتمد للتكوين، كما أدى احتساب زيادة اللزوجة وإجهاد الخضوع إلى خفض الهامش المحسوب من 5.78 إلى 5.44 رطل/بوصة². وتُبرز النتائج أهمية دقة حساب فواقد الاحتكاك في تقييم هامش الضغط وإدارة الحد العلوي لنفاذة الحفر الآمنة، خاصة في التكوينات الكربوناتيّة المتشققة، بما يدعم تقييم مخاطر دخول سائل الحفر إلى الشقوق القائمة وتفاقم فقدان الدوران.

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1. Introduction

Pressure losses in a drilling circulation system act at distinct locations along the flow circuit. Standpipe pressure (SPP) sums frictional losses across surface lines, the drill string, bit nozzles, and the return annulus, whereas the circulating load on an exposed formation horizon equals hydrostatic head plus the annular friction accumulated along the return path above that depth (Bourgoyne et al., 1986). Consequently, a small annular fraction of SPP can coexist with a narrow

downhole formation-pressure reserve—a critical distinction in naturally fractured carbonates, where drilling fluid can invade pre-existing conductive fractures without requiring intact-rock tensile breakdown (Lavrov, 2016).

Classical work established laminar Bingham solutions for pipes, plane slots, and concentric annuli (Laird, 1957; Fredrickson & Bird, 1958; Melrose et al., 1958), with analytical extensions for wall slip (Gryparis & Georgiou, 2022). Later drilling-hydraulics studies addressed constitutive calibration, eccentric and Herschel–Bulkley annular flow (Kelessidis et al., 2011; Ashena et al., 2021; Saasen et al., 2021; Ashena, 2025, corrected in Ashena, 2026), and hole-cleaning effects on ECD (Badrouchi et al., 2022; Al-Rubaii et al., 2023). Yet an error reported against total circulation pressure does not show whether it exhausts the reserve left at an exposed formation. Here, analytical truncation and curvature discrepancies are evaluated against that local reserve and translated into model-specific critical formation limits ($F_{crit,m}$) and open disagreement intervals across field operating states from Belhedan wells V33, V36, and V39 (Kaukau, 2022), a 1,860-state dimensionless survey, and solids-loading sensitivities to delimit where model choice changes the margin classification.

2. Field Context and Operational Data

2.1 Operational Baselines and Published Loss Horizons

Kaukau (2022) documented severe lost circulation and remedial cement-plug treatments in the 12.25-in carbonate sections of Belhedan wells V33, V36, and V39. Table 1 distinguishes four provenance categories: (i) actual field operating inputs from the Belhedan drilling records; (ii) field-based WellPlan hydraulic model outputs (Halliburton, 2024) calculated from those inputs, rather than measured independently downhole; (iii) adopted formation-equivalent assessment boundaries F of 9.48 ppg (V33) and 9.50 ppg (V36 and V39), used as upper assessment boundaries rather than documented LOT/FIT results; and (iv) parametric cuttings and rheological sensitivities around the field operating states. V33 is evaluated at the open-ended pipe depth for cement-plug placement (3,846 ft), and V36 and V39 at the published total-loss onset depths (3,724 and 3,677 ft) under full returns. Each vertical column comprises a 2,000-ft cased drill-pipe annulus inside 13-3/8-in casing (12.415-in ID), an open-hole drill-pipe section of length $z - 2,300$ ft (12.250-in hole), and a 300-ft drill-collar interval.

Table 1. Published loss-history anchors (Kaukau, 2022), actual field operating parameters, adopted formation-equivalent assessment boundaries, and WellPlan baselines.

Parameter / Operational Metric	V33	V36	V39
Evaluation depth z (ft) [TVD]	3,846	3,724	3,677
Loss-history anchor meaning (Kaukau, 2022)	Plug-placement depth	Total-loss onset	Total-loss onset
Remedial cement plugs / loss NPT (h)	5 / 111.5	6 / 255.5	2 / 61.0
Cased DP / open-hole DP / collar lengths (ft)	2,000 / 1,546 / 300	2,000 / 1,424 / 300	2,000 / 1,377 / 300
13-3/8-in casing ID / open-hole diameter (in)	12.415 / 12.250	12.415 / 12.250	12.415 / 12.250
Drill-pipe OD / drill-collar OD (in)	5.000 / 8.000	5.000 / 8.000	5.000 / 8.000
Baseline mud density MW (ppg) / PV (cP)	9.20 / 15	9.20 / 15	9.20 / 15
Yield point YP (lbf/100 ft ²)	12	14	16
Circulation rate Q (US gal/min)	400	450	450
Adopted formation-equivalent boundary F (ppg)	9.48	9.50	9.50

Bit nozzle diameters (three nozzles, in)	16/32 each	16/32 each	14/32 each
WellPlan standpipe / bit / annular ΔP (psi)	809.77 / 390.62 / 30.92	995.22 / 494.37 / 34.94	1,349.18 / 843.38 / 39.26

2.2 Physical Interpretation of the Pressure Boundary

All three wells share identical annular diameters, a 9.20-ppg mud weight, and a 15-cP plastic viscosity, while circulation rates (400–450 gpm) and yield points (12–16 lbf/100 ft²) produce distinct plug fractions. In a connected natural fracture of aperture b and invaded length ℓ , Bingham fluid invasion begins when $\Delta P_{\text{inv}} > 2\tau_y \ell / b$, whereas reopening a closed fracture depends on effective normal stress and fracture orientation (Lavrov, 2016). At the adopted assessment boundaries F of 9.48, 9.50, and 9.50 ppg, the initial clean-mud pressure allowances above static hydrostatic pressure are 56.00, 58.09, and 57.36 psi. For any added solids volume fraction C_v and model friction loss $\Delta P_{a,m}(C_v)$, the critical equivalent-density boundary is:

$$F_{\text{crit},m}(C_v) = MW + C_v(\rho_s - MW) + \frac{\Delta P_{a,m}(C_v)}{0.052z}.$$

Any specified boundary F yields a positive pressure reserve exactly when $F > F_{\text{crit},m}$. While the positive or negative margin classification depends on F , both the pressure differences between friction models and the critical limits $F_{\text{crit},m}(C_v)$ are fixed by the hydraulic state and can be compared directly against any alternative boundary without repeating the friction calculations.

3. Hydraulic and Sensitivity Models

3.1 Sectional Geometry and Laminar Regime Screen

For annular section k with outer diameter $D_{o,k}$ and inner diameter $D_{i,k}$, the flow area A_k , mean axial velocity v_k , and hydraulic diameter $d_{h,k}$ are:

$$A_k = \frac{\pi(D_{o,k}^2 - D_{i,k}^2)}{4}, \quad v_k = \frac{Q}{A_k}, \quad d_{h,k} = D_{o,k} - D_{i,k} \quad (1)$$

Exact-slot and concentric-annulus calculations use SI units (1 ft = 0.3048 m, 1 US gal/min = $6.30901964 \times 10^{-5}$ m³/s, 1 psi = 6894.757293 Pa, $\mu_p = 10^{-3}$ PV Pa·s, and $\tau_y = 0.4788026$ YP Pa) before summing sectional losses in psi. The flow regime is screened using the field-unit Bingham Reynolds number (with v in ft/s and d_h in inches):

$$Re_{\text{screen}} = \frac{757MW v d_h}{PV + 5YP d_h / v} \quad (2)$$

Equation (2) gives sectional values of 178.25, 189.60, and 376.38 for V33; 193.65, 206.00, and 409.77 for V36; and 170.28, 181.19, and 362.96 for V39 (in cased drill-pipe, open-hole drill-pipe, and drill-collar order), supporting the laminar-flow assumption for the baseline operating states.

3.2 Pipe-Equivalent and Plane-Slot Formulations

The engineering pipe-equivalent correlation substitutes hydraulic diameter into the two-term Bingham pipe relation (Bourgoyne et al., 1986; Melrose et al., 1958), with $g_{PE,k}$ in psi/ft, v_k in ft/s, $d_{h,k}$ in inches, PV in cP, and YP in lbf/100 ft²:

$$g_{PE,k} = \frac{PV v_k}{1500 d_{h,k}^2} + \frac{YP}{225 d_{h,k}}, \quad \Delta P_{a,PE} = \Sigma g_{PE,k} L_k \quad (3)$$

For an area-preserving plane slot of half-gap $H_k = (D_{o,k} - D_{i,k})/4$, the exact Buckingham–Reiner slot relation between mean velocity v and positive frictional pressure gradient G_s is (Laird, 1957; Fredrickson & Bird, 1958; Melrose et al., 1958):

$$G_s \left[1 - \frac{3\xi}{2} + \frac{\xi^3}{2} \right] = \frac{3\mu_p v}{H^2}, \quad \xi = \frac{\tau_y}{G_s H}, \quad 0 < \xi < 1 \quad (4)$$

Solving Equation (4) for the yielded root $G_s > \tau_y/H$ retains the cubic plug-flow term, whereas omitting $\xi^3/2$ yields the conventional linearized slot approximation:

$$G_{lin} = \frac{3\mu_p v}{H^2} + \frac{3\tau_y}{2H}, \quad \Delta P_{a,m} = \Sigma G_{m,k} L_k \quad (5)$$

3.3 Concentric Annular Solution and Analytical Verification

The concentric model solves axial momentum and Bingham constitutive relations with no wall slip between inner radius r_i and outer radius r_o (Laird, 1957; Fredrickson & Bird, 1958).

In terms of the zero-shear radius r_0 , axial pressure gradient G , and signed shear rate $s(r) = du/dr$:

$$\tau(r) = \frac{G}{2} \left(\frac{r_0^2}{r} - r \right), \quad s(r) = \frac{\text{sgn}(\tau) \max(|\tau| - \tau_y, 0)}{\mu_p} \quad (6)$$

No-slip boundary conditions at both walls and the imposed volumetric flow rate Q require simultaneously:

$$\int_{r_i}^{r_o} s(r) dr = 0, \quad Q = \pi \int_{r_i}^{r_o} (r_o^2 - r^2) s(r) dr \quad (7)$$

With $a = \tau_y/G$, the plug boundaries are $r_{p1} = \sqrt{(r_o^2 + a^2)} - a$ and $r_{p2} = \sqrt{(r_o^2 + a^2)} + a$. Integration by parts together with wall-velocity closure yields the closed-form no-slip Bingham annular throughput (Fredrickson & Bird, 1958; Gryparis & Georgiou, 2022):

$$Q = \frac{\pi G}{8\mu_p} \left[r_o^4 - r_i^4 - 2r_o^2(r_o^2 - r_i^2) - \frac{8a}{3}(r_o^3 + r_i^3) + \frac{16a}{3}(r_o^2 + a^2)^{3/2} \right]. \quad (8)$$

Independent numerical quadrature of the piecewise yielded shear-rate profile in Equation (7) was used to verify the concentric-annulus solution against the imposed flow rate and no-slip conditions. For each section and trial gradient $G > \tau_y/H$, bracketing r_0 within $[\sqrt{(r_i(r_i + 2a))}, \sqrt{(r_o(r_o - 2a))}]$ from the no-slip closure and solving Equation (8) reduced both the solver relative throughput residual $|Q_{\text{calc}} - Q|/Q$ and the relative difference from numerical quadrature below 10^{-12} , supporting all five reported decimal places. Using unrounded SI inputs, the resulting friction losses

were 27.88325, 31.42041, and 34.94732 psi for V33, V36, and V39, respectively (Table 2), with V39 sectional losses of 17.42506, 12.33872, and 5.18354 psi (34.95 psi after rounding). The zero-yield-stress limit ($\tau_y \rightarrow 0$) recovered the analytical Newtonian annular solution.

3.4 Downhole Pressure Budget, Bit Orifice Check, and Solids Loading

At evaluation depth z under full annular returns, the initial pressure allowance B , remaining reserve R_m , equivalent circulating density ECD_m , and friction utilization percentage U_m for model m are:

$$B = 0.052z(F - MW), \quad R_m = B - \Delta P_{a,m} \quad (9)$$

$$ECD_m = MW + \frac{\Delta P_{a,m}}{0.052z}, \quad U_m = 100 \frac{\Delta P_{a,m}}{B} \quad (10)$$

Surface backpressure is zero, and SPP is partitioned separately from the downhole budget. An orifice check across the three bit nozzles of integer 32nd-inch designation d and total flow area TFA (in²) verifies the surface-to-bit partition:

$$TFA = \frac{3\pi}{4} \left(\frac{d}{32}\right)^2, \quad \Delta P_{bit} = \frac{8.311 \times 10^{-5} MW Q^2}{C_d^2 TFA^2}, \quad C_d = 0.95. \quad (11)$$

For a uniform added suspended-solids volume fraction C_v of particle density $\rho_s = 8.345404$ SG_s (ppg), volume additivity at fixed baseline friction gives the hydrostatic increment, remaining margin, and zero-margin critical concentration:

$$\Delta \rho = C_v(\rho_s - MW), \quad R_m(C_v) = R_m - 0.052z\Delta \rho \quad (12)$$

$$C_{v,crit,m} = \frac{R_m}{0.052z(\rho_s - MW)} \quad (13)$$

Here SG_s = 2.145 is treated as a low-density diagnostic sensitivity case, not a measured cuttings density for these wells. By contrast, SG_s = 2.71 and 2.86 are mineral-density references for calcite and dolomite, respectively (Mineral Data Publishing, 2005a, 2005b). At fixed solids concentration, denser particles raise hydrostatic loading and shift both the zero-margin solids threshold and the formation-limit window in which hydraulic models disagree (Badrouchi et al., 2022).

3.5 Model-Disagreement Interval, Stress Tests, and Transport Screening

Given the minimum (P_{min}) and maximum (P_{max}) baseline annular losses among the compared hydraulic formulations, the concentration interval over which models disagree on the margin sign is bounded by:

$$C_{low} = \frac{B - P_{max}}{0.052z(\rho_s - MW)}, \quad C_{high} = \frac{B - P_{min}}{0.052z(\rho_s - MW)} \quad (14)$$

A deterministic perturbation test simultaneously reduces F by $\delta F = 0.05$ ppg and increases the reference WellPlan friction $\Delta P_{a,ref}$ by $\varepsilon = 0.10$:

$$R_{pert} = B - (1 + \varepsilon)\Delta P_{a,ref} - 0.052z[\delta F + C_v(\rho_s - MW)] \quad (15)$$

Cuttings transport is screened using Robinson and Morgan's (2004) empirical carrying-capacity index (CCI) in field units (with $\theta_{300} = PV + YP$, $\theta_{600} = 2PV + YP$, and velocity v_{fpm} in ft/min; Al-Rubaii et al., 2023):

$$n = \frac{\ln(\theta_{600}/\theta_{300})}{\ln 2}, \quad K = 511^{1-n} \theta_{300}, \quad CCI = \frac{MW v_{fpm} K}{400000} \quad (16)$$

3.6 Dimensionless Truncation and Curvature Criteria

Using the radius ratio $\kappa = r_i/r_o = D_i/D_o$ and Bingham number $Bi = \tau_y H / (\mu_p v)$ with $H = (r_o - r_i)/2$ and $f(\xi) = 1 - 3\xi/2 + \xi^3/2$, Equation (4) rearranges to:

$$Bi = \frac{3\xi}{f(\xi)}, \quad \frac{G_{lin}}{G_s} = f(\xi) + \frac{3\xi}{2} = 1 + \frac{\xi^3}{2}. \quad (17)$$

Consequently, the fixed-flow relative pressure-gradient error of the linearized slot is $E_p = (G_{lin} - G_s)/G_s = \xi^3/2$, increasing monotonically to 50% as $\xi \rightarrow 1$. An admissible relative error $e < 0.5$ requires $\xi \leq (2e)^{1/3}$ and the corresponding upper Bi from Equation (17). Along a segmented well column, the overall relative truncation error is the exact-slot loss-weighted average:

$$E_{P,column} = \frac{\sum_k G_{s,k} L_k \xi_k^3/2}{\sum_k G_{s,k} L_k}. \quad (18)$$

This fixed-flow pressure error differs from the fixed-gradient velocity error $E_v = \xi^3/[2f(\xi)]$, where the linearized velocity turns negative for $\xi > 2/3$ whereas fixed-flow inversion remains positive but overpredicts gradient.

To isolate annular curvature in dimensionless form, set $r_o = 1$, $v = 1$, and $\mu_p = 1$, giving $H = (1 - \kappa)/2$, $\tau_y = Bi/H$, and $Q = \pi(1 - \kappa^2)$. Solving Equation (8) for the dimensionless annular gradient G_a defines the curvature excess E_c and combined linearized-to-annular error $E_{lin,a}$:

$$E_c(\kappa, Bi) = \frac{G_s}{G_a} - 1, \quad E_{lin,a} = \left(1 + \frac{\xi^3}{2}\right)(1 + E_c) - 1. \quad (19)$$

3.7 Coupled Suspension Rheology and Residual-Margin Criterion

To couple solids density with suspension rheology at fixed flow rate, the baseline mud is treated as the suspending medium for newly added noncolloidal particles of volume fraction C_v . Effective plastic viscosity follows the Krieger and Dougherty (1959) closure:

$$f_\mu(C_v) = \left(1 - \frac{C_v}{\phi_m}\right)^{-[\eta]\phi_m}, \quad [\eta] = 2.5, \quad \phi_m = 0.60, \quad \mu_{p,eff} = \mu_p f_\mu. \quad (20)$$

The isotropic homogenization closure of Chateau et al. (2008) scales yield stress by $\tau_{y,eff}/\tau_y = \sqrt{(1 - C_v)f_\mu}$, which is compared against viscosity-only (constant τ_y) and equal-factor (τ_y scaled by f_μ) sensitivity cases:

$$R_{m,coupled} = B - \Delta P_{a,m}(\mu_{p,eff}, \tau_{y,eff}) - 0.052zC_v(\rho_s - MW). \quad (21)$$

Equal scaling of both parameters leaves Bi invariant and multiplies gradient by f_μ , whereas micromechanical coupling scales Bi by $\sqrt{(1 - C_v)/f_\mu}$. Differentiating Equation (4) at fixed velocity gives the logarithmic parameter sensitivities (whose sum is unity):

$$\frac{\partial \ln G_s}{\partial \ln \mu_p} = \frac{f(\xi)}{1 - \xi^3}, \quad \frac{\partial \ln G_s}{\partial \ln \tau_y} = \frac{3\xi(1 - \xi^2)}{2(1 - \xi^3)}. \quad (22)$$

To link hydraulic model discrepancy directly to margin classification, let $P_a(C_v)$ denote the concentric-annulus friction loss and $P_m(C_v)$ a comparison-model loss at the same state, with $\Delta P_m = P_m - P_a$ and concentric reserve R_a . For $R_a > 0$ and $\Delta P_m \geq 0$, the residual-margin ratio $\Lambda_m = \Delta P_m / R_a$ quantifies the fraction of the remaining reserve consumed by the model discrepancy:

$$R_m = R_a - \Delta P_m. \quad (23)$$

$$\Lambda_m = \Delta P_m / R_a; \quad F_{\text{crit},a} < F < F_{\text{crit},m} \quad \text{when } \Delta P_m > 0. \quad (24)$$

A comparison model preserves a positive reserve for $\Lambda_m < 1$ ($F > F_{\text{crit},m}$), reaches zero reserve at the endpoint $\Lambda_m = 1$ ($F = F_{\text{crit},m}$), and reverses the margin sign for $\Lambda_m > 1$ —equivalently, whenever F lies strictly within the open disagreement interval $F_{\text{crit},a} < F < F_{\text{crit},m}$ of width $\Delta P_m / (0.052z)$ —while satisfying a prescribed reserve tolerance α when $\Lambda_m \leq \alpha$.

4. Results

4.1 Annular Friction Models and Downhole Pressure Budgets

Table 2 and Figure 1 compare the five hydraulic formulations alongside the baseline downhole pressure budgets. Across all three wells, the exact plane-slot and concentric-annulus losses differ by only 0.10–0.12 psi (0.34–0.37%), while the pipe-equivalent correlation tracks the field WellPlan baselines within 0.24–0.30 psi (0.75–0.78%). Because the sectional plug ratios ξ span 0.704–0.822, omitting the cubic plug term $\xi^3/2$ in the linearized slot adds 6.97–9.24 psi (24.91–26.36%) above the exact slot and 7.07–9.36 psi above the concentric solution. Annular friction consumes 55.22–68.44% of the initial downhole formation allowance B but only 2.91–3.82% of SPP (Figure 2). Evaluating Equation (11) gives bit pressure drops of 390.67, 494.44, and 843.50 psi for V33, V36, and V39, within 0.05–0.12 psi of the WellPlan bit baselines, confirming that the 353.96-psi SPP

difference between V36 and V39 stems primarily from switching 16/32-in to 14/32-in nozzles rather than increased annular loading.

Table 2. Calculated annular frictional pressure losses and reference downhole pressure budgets at the evaluation horizons.

Hydraulic Formulation / Budget Metric	V33	V36	V39
Concentric annulus ΔP_a (psi)	27.88	31.42	34.95
Exact plane slot ΔP_a (psi)	27.99	31.53	35.07
Pipe-equivalent correlation ΔP_a (psi)	30.68	34.67	38.96
WellPlan baseline $\Delta P_{a,\text{ref}}$ (psi)	30.92	34.94	39.26
Linearized plane slot ΔP_a (psi)	34.96	39.48	44.31

Initial formation allowance B (psi) / Reference R (psi)	56.00 / 25.08	58.09 / 23.15	57.36 / 18.10
Reference ECD (ppg) / Allowance used U (%) / ($\Delta P_{a,ref}$/SPP %)	9.355 / 55.22 / 3.82	9.380 / 60.14 / 3.51	9.405 / 68.44 / 2.91

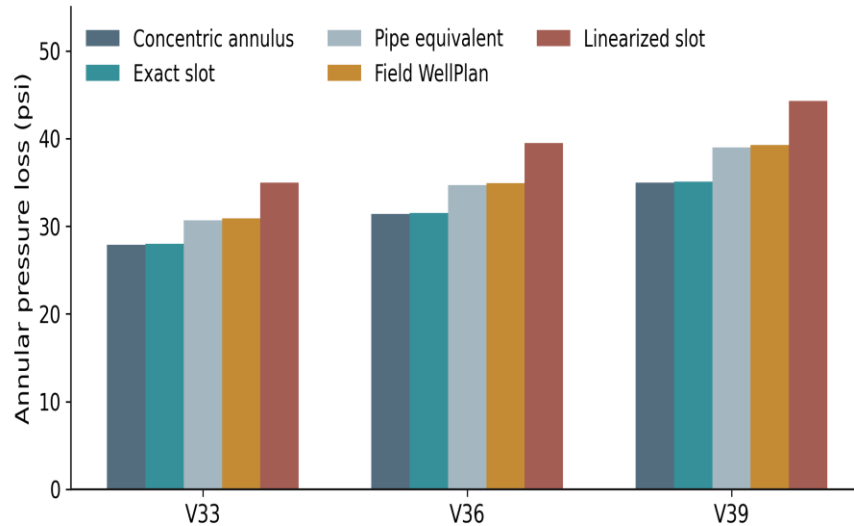


Figure 1. Annular frictional pressure-loss comparison across the three Belhedan well operating states.

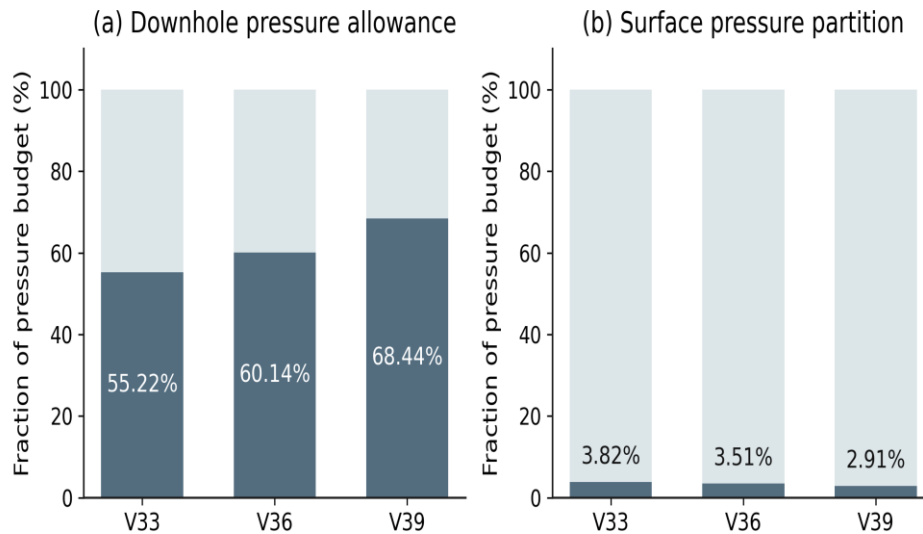


Figure 2. Reference annular friction as a share of (a) downhole pressure allowance B and (b) standpipe pressure SPP.

4.2 Cuttings Density, Coupled Suspension Rheology, and Margin Reversal

At the low-density diagnostic value $SG_s = 2.145$, adding 1 vol% solids raises mud weight by 0.0870 ppg and hydrostatic pressure by 17.40, 16.85, and 16.64 psi in V33, V36, and V39; with baseline friction held fixed, V39 retains positive reserves under the concentric (+5.78 psi), exact-slot (+5.66 psi), pipe-equivalent (+1.76 psi), and WellPlan (+1.46 psi) models, but yields a

negative margin (-3.59 psi) under the linearized slot (Figure 3a). At the calcite-reference density ($SG_s = 2.71$, adding 0.1342 ppg or 26.83 , 25.98 , and 25.65 psi), the concentric model retains small positive margins in V33 ($+1.28$ psi) and V36 ($+0.69$ psi; $+1.18$ and $+0.58$ psi for the exact slot), whereas WellPlan (-1.75 and -2.83 psi) and the linearized slot (-5.79 and -7.37 psi) predict negative values under the same fixed-friction assumptions, while V39 is negative across all models (-3.24 psi concentric, -7.55 psi WellPlan, and -12.60 psi linearized slot; Figure 3b). At the dolomite-reference density ($SG_s = 2.86$), all three wells turn negative (-1.22 , -1.73 , and -5.63 psi concentric). Model-dependent margin classification thus occurs in V33 and V36 at the calcite reference density as well as in V39 at $SG_s = 2.145$, although small positive reserves do not by themselves establish safe operation; Table 3 compiles the zero-margin concentrations and 1 vol% micromechanical critical limits $F_{crit,micro}$.

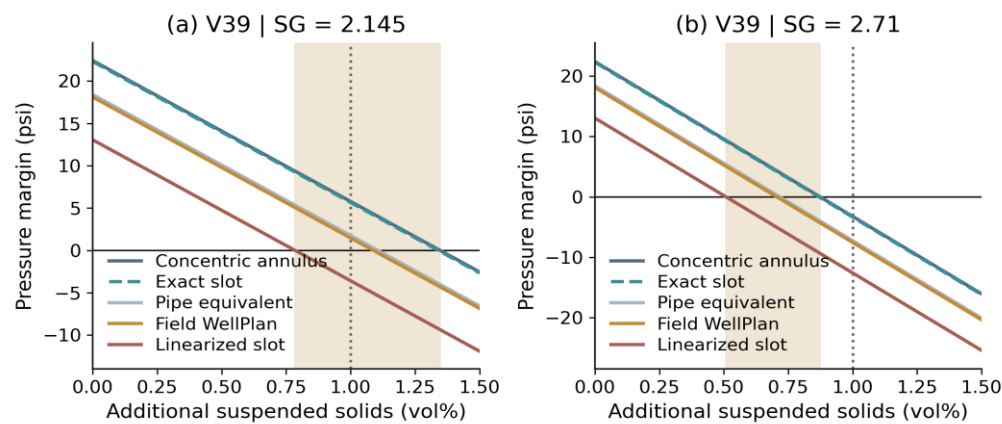


Figure 3. V39 pressure-margin sensitivity to added solids at fixed friction for (a) $SG_s = 2.145$ and (b) $SG_s = 2.71$ (shaded disagreement zone).

Table 3. Zero-margin solids concentrations (vol%) across fixed-friction models and coupled concentric closures, with micromechanical F_{crit} .

Well	SG_s	Linearized (fixed)	WellPlan (fixed)	Concentric (fixed)	Concentric (micro)	Concentric (equal)	$F_{crit,micro}$ at 1 vol% (ppg)
V33	2.145	1.209	1.441	1.616	1.591	1.551	9.42779
V33	2.710	0.784	0.935	1.048	1.037	1.021	9.47494
V33	2.860	0.717	0.855	0.958	0.950	0.936	9.48746
V36	2.145	1.105	1.374	1.583	1.555	1.510	9.45084
V36	2.710	0.716	0.891	1.027	1.015	0.996	9.49799
V36	2.860	0.655	0.815	0.939	0.929	0.913	9.51051
V39	2.145	0.784	1.088	1.347	1.321	1.278	9.47153
V39	2.710	0.509	0.706	0.874	0.863	0.844	9.51868
V39	2.860	0.465	0.645	0.799	0.790	0.775	9.53120

4.3 Deterministic Stress Tests and Empirical Transport Screening

The deterministic perturbation tests in Table 4 show that lowering F by 0.05 ppg or increasing reference friction by 10% individually preserves positive clean-fluid margins across all three wells, whereas combining both perturbations reduces the V39 reserve to 4.62 psi and adding 1 vol% solids at $SG_s = 2.145$ drives all three wells across the zero-margin threshold (-5.41

to -12.02 psi). Empirical transport screening (Table 5 and Figure 4b) shows that while drill-collar CCI values in V36 (1.03) and V39 (1.34) exceed 1.0, cased and open-hole drill-pipe CCI values remain between 0.45 and 0.92. Drill-pipe CCI values below 1.0 indicate conditions that warrant further hole-cleaning assessment rather than direct evidence of measured cuttings accumulation.

Table 4. Deterministic stress tests on remaining pressure reserve using the reference WellPlan friction baseline (psi).

Well	F reduced by 0.05 ppg	Friction increased 10%	Both perturbations	Both + 1 vol% solids (SG _s 2.145)
V33	15.08	21.99	11.99	-5.41
V36	13.47	19.66	9.98	-6.87
V39	8.54	14.18	4.62	-12.02

Table 5. Empirical carrying-capacity index (CCI) transport screening for the baseline well states.

Well	Flow index n	Consistency K (field units)	Cased DP CCI	Open-hole DP CCI	Drill-collar CCI
V33	0.637	259.0	0.45	0.47	0.68
V36	0.601	348.2	0.68	0.71	1.03
V39	0.569	454.7	0.89	0.92	1.34

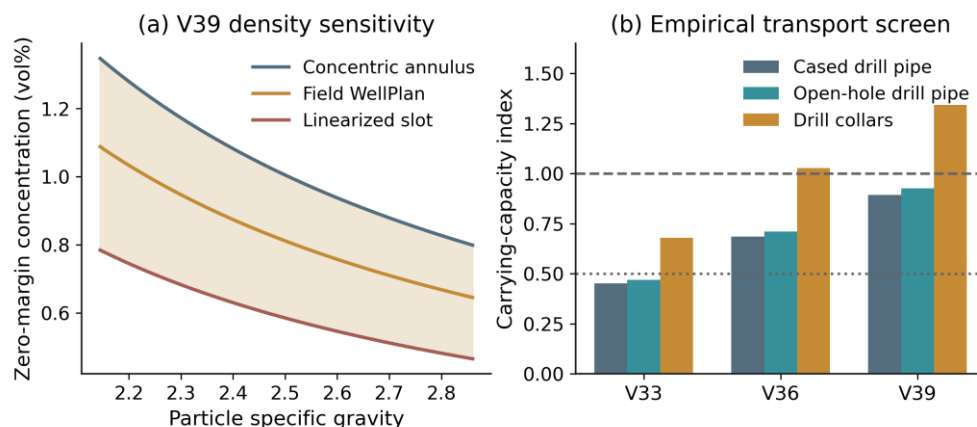


Figure 4. (a) Particle-density dependence of V39 zero-margin concentration; (b) sectional CCI values against 0.5 and 1.0 thresholds.

4.4 Dimensionless Error Map, Rheological Coupling Increments, and Residual-Margin Test

From Equation (17), restricting the fixed-flow linearized-slot truncation error $\xi^3/2$ to 1%, 5%, 10%, and 20% requires plug ratios of $\xi \leq 0.271442$, 0.464159, 0.584804, and 0.736806 ($Bi \leq 1.35082$, 3.93620, 7.87456, and 23.31898), respectively. Because the nine Belhedan well sections operate at $Bi = 17.86$ – 55.42 ($\xi = 0.704$ – 0.822), all nine lie above the 5% and 10% limits, producing column-weighted truncation errors of 24.91–26.36% via Equation (18). Conversely, across the 1,860-case dimensionless curvature survey ($0.20 \leq \kappa \leq 0.80$ in steps of 0.02; $1 \leq Bi \leq 60$ in unit steps, with a separately evaluated Newtonian reference limit $Bi = 0$ in Table 6), the largest sampled slot-to-annulus curvature error within the surveyed Bingham domain is $E_c = 3.3589\%$ at $Bi = 1$ and $\kappa = 0.20$ (3.9496% at the Newtonian limit), falling to 0.127–0.412% across the nine well

sections (Figure 5). An independent node-by-node recalculation reproduced the tabulated dimensionless values and located the same 3.3589% maximum.

Table 6. Exact-slot excess over concentric-annulus pressure gradient E_c (%) at selected nodes of the 1,860-state dimensionless survey.

$Bi \setminus \kappa$	0.20	0.30	0.40	0.50	0.60	0.70	0.80
0 (Newtonian)	3.9496	2.2966	1.3585	0.7872	0.4309	0.2111	0.0828
1	3.3589	1.9629	1.1644	0.6759	0.3704	0.1815	0.0713
5	2.4957	1.4726	0.8784	0.5116	0.2809	0.1379	0.0542
10	2.0453	1.2138	0.7264	0.4239	0.2331	0.1145	0.0450
20	1.6111	0.9616	0.5774	0.3377	0.1859	0.0914	0.0359
40	1.2300	0.7379	0.4444	0.2604	0.1435	0.0706	0.0278
60	1.0388	0.6248	0.3768	0.2210	0.1219	0.0600	0.0236

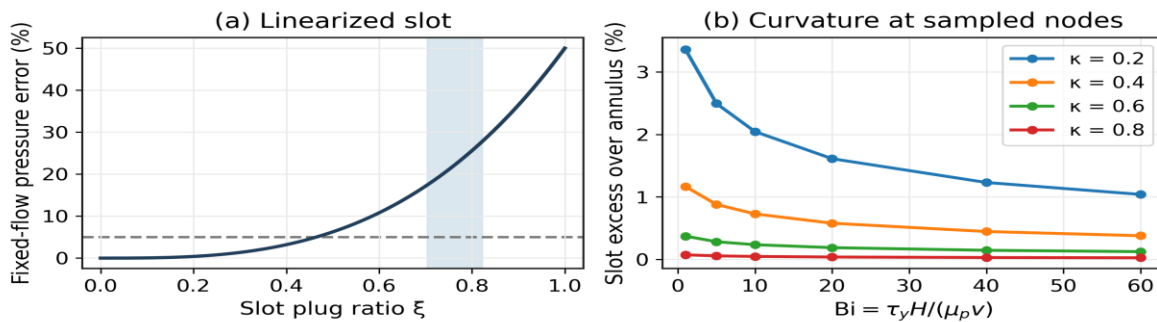


Figure 5. (a) Analytical fixed-flow linearized-slot truncation error (well ξ range shaded); (b) curvature error E_c across Bi and κ .

At 1 vol% added solid, Equation (20) raises plastic viscosity by 2.5531%, while the Chateau et al. (2008) homogenization raises yield stress by 0.7609%. Table 7 shows that suspension coupling adds 0.085–0.892 psi to concentric friction at 1 vol% (lowering the V39 reserve at $SG_s = 2.145$ from +5.78 to +5.44 psi under micromechanical coupling and +4.89 psi under equal-factor scaling). In V36 at $SG_s = 2.71$, equal-factor coupling shifts the zero-margin concentration from 1.027 to 0.996 vol% (Table 3), reversing the concentric margin sign at 1 vol%. Clean-fluid concentric critical limits $F_{crit,a}(0)$ are 9.33942, 9.36226, and 9.38278 ppg for V33, V36, and V39, with a boundary sensitivity of 0.052z (1.91–2.00 psi per 0.01 ppg). Table 8 applies the residual-margin criterion at 1 vol% and $SG_s = 2.145$: the linearized-slot excess consumes 66.0% ($A_{lin} = 0.660$), 82.0% ($A_{lin} = 0.820$), and 162.1% ($A_{lin} = 1.621$) of the concentric reserve R_a in V33, V36, and V39. Opposite margin signs ($R_a > 0$ and $R_{lin} < 0$) arise strictly within the open interval $F_{crit,a} < F < F_{crit,lin}$ —(9.42643, 9.46180) ppg for V33, (9.44926, 9.49089) ppg for V36, and (9.46978, 9.51875) ppg for V39 (enclosing $F = 9.50$ ppg in V39)—whereas at the endpoints $F = F_{crit,a}$ or $F = F_{crit,lin}$ the corresponding model reserve is zero. These open intervals, rather than a single assumed formation limit, provide transferable hydraulic screening boundaries.

Table 7. Additional concentric-annulus friction caused by suspension rheological coupling (psi) at 0.5, 1.0, and 2.0 vol% solids.

Well	Added solids (vol%)	Viscosity-only ΔP (psi)	Micromechanical ΔP (psi)	Equal-factor ΔP (psi)
V33	0.5 / 1.0 / 2.0	0.042 / 0.085 / 0.173	0.135 / 0.272 / 0.552	0.352 / 0.712 / 1.455
V36	0.5 / 1.0 / 2.0	0.047 / 0.094 / 0.191	0.151 / 0.305 / 0.620	0.397 / 0.802 / 1.639
V39	0.5 / 1.0 / 2.0	0.048 / 0.097 / 0.198	0.166 / 0.334 / 0.680	0.441 / 0.892 / 1.823

Table 8. Residual-margin classification test and critical formation-limit intervals at 1 vol% added solid and $SG_s = 2.145$.

Well	R_a (psi)	ΔP_{slot} (psi)	ΔP_{lin} (psi)	$A_{\text{lin}} = \Delta P_{\text{lin}}/R_a$	$F_{\text{crit},a}$ (ppg)	$F_{\text{crit},lin}$ (ppg)
V33	10.71	0.103	7.07	0.660	9.42643	9.46180
V36	9.82	0.115	8.06	0.820	9.44926	9.49089
V39	5.78	0.121	9.36	1.621	9.46978	9.51875

5. Discussion

5.1 Separating Plug Truncation from Annular Curvature in Narrow-Margin Decisions

The residual reserve determines whether a friction-model discrepancy changes the margin classification. For a positive concentric reserve ($R_a > 0$) and nonnegative excess loss ($\Delta P_m \geq 0$), $A_m = \Delta P_m/R_a$ is an algebraic normalization of the pressure budget rather than a new physical law, linking model error directly to the remaining formation allowance. Table 8 gives the equivalent criterion through model-specific critical limits: two formulations predict opposite margin signs strictly within the open interval $F_{\text{crit},a} < F < F_{\text{crit},m}$, whereas at either endpoint one reserve is zero. The dimensionless results explain the source of that discrepancy: plug-term truncation dominates curvature error over the investigated well conditions. Mapping a concentric annulus to an area-preserving slot introduces at most 3.3589% curvature error over the 1,860 sampled Bingham states (and 0.127–0.412% across the well sections, vs. 3.9496% at the Newtonian limit $Bi = 0$), whereas omitting the cubic plug term $\xi^3/2$ introduces more than 20% pressure-gradient error once $\xi > 0.736806$. Retaining the full slot relation removes this truncation error without solving the annular integral for every screening case.

The close agreement between the pipe-equivalent correlation and the field-based WellPlan reference (0.24–0.30 psi) is a consistency check between hydraulic calculations rather than independent downhole validation. Pipe eccentricity may reduce annular friction as flow shifts toward the wider gap, depending on geometry and rheology (Kelessidis et al., 2011). An imposed 10% reduction in V39 concentric friction increases its reserve at 1 vol% and $SG_s = 2.145$ from +5.78 to +9.27 psi, although slower flow in the narrow gap can still impair hole cleaning (Badrouchi et al., 2022). Extending this verification to Herschel–Bulkley fluids requires rheological calibration across the relevant shear-rate range and verified annular flow relations (Saasen et al., 2021; Ashena, 2025, corrected in Ashena, 2026).

5.2 Joint Role of Cuttings Mineralogy, Rheological Coupling, and Hole Cleaning

Cuttings density raises the hydrostatic load, while changes in suspension rheology modify annular friction. At the V39 assessment boundary of $F = 9.50$ ppg and 1 vol% solids, inverting the fixed-friction pressure budget predicts opposite margin signs over $1.920 < SG_s < 2.507$ (approximately $1.92 < SG_s < 2.51$, with WellPlan crossing zero at $SG_s = 2.237$). The selected $SG_s = 2.145$ lies inside this interval and serves as a diagnostic sensitivity case, not a measured mineralogical value.

Replacing $SG_s = 2.145$ with calcite ($SG_s = 2.71$) or dolomite ($SG_s = 2.86$) reference densities adds 9.02–9.43 psi or 11.41–11.93 psi to the hydrostatic column, comparable to the 7.07–9.36 psi gap between linearized-slot and concentric friction estimates. At fixed friction, increasing particle density shifts every critical limit $F_{crit,m}$ upward by $\Delta\rho$ without changing the inter-model width $\Delta P_m/(0.052z)$. At the calcite-reference density ($SG_s = 2.71$, where $\Delta\rho$ increases by 0.04715 ppg), V39's disagreement interval shifts to (9.51694, 9.56591) ppg—above $F = 9.50$ ppg, so all models yield negative margins (–3.24 to –12.60 psi)—whereas the V33 and V36 intervals shift to (9.47358, 9.50895) and (9.49642, 9.53804) ppg (with WellPlan limits at 9.48877 and 9.51459 ppg), straddling $F = 9.48$ and 9.50 ppg. Because a 0.01-ppg change in F alters the reserve by 1.91–2.00 psi—exceeding the +1.28 and +0.69 psi concentric margins in V33 and V36—small-margin signs are sensitive to F and are directly bounded by $F_{crit,m}$. At $SG_s = 2.86$, $F_{crit,a}$ exceeds F in all three wells.

Rheological coupling also matters near zero reserve. The sensitivities in Equation (22) show that pressure loss in the investigated high-plug-ratio states ($\xi = 0.704$ –0.822) responds more strongly to yield-stress changes than to viscosity changes alone: at 1 vol% solids, viscosity-only scaling adds 0.085–0.097 psi, micromechanical coupling adds 0.272–0.334 psi, and equal-factor scaling adds 0.712–0.892 psi (Table 7). For V36 at the calcite-reference density ($SG_s = 2.71$), equal-factor scaling moves the zero-margin concentration from 1.027 to 0.996 vol% (Table 3), crossing the 1 vol% test case (+0.69 to –0.11 psi). These closures represent theoretical sensitivity bounds rather than laboratory measurements on field cuttings, and drill-pipe CCI values below 1.0 (0.45–0.92 in Table 5) warrant separate hole-cleaning assessment rather than measuring retained cuttings or establishing a single cause of historical fluid losses.

5.3 Scope and Field Application

The calculations represent a full-return hydraulic reference state based on actual field operating inputs and WellPlan outputs, assuming vertical, in-gauge, concentric annuli, stationary pipe, and no wall slip while varying solids loading, particle density, and suspension rheology parametrically. Relating the adopted assessment boundaries F to actual fracture invasion, reopening, or leak-off would require downhole pressure and fracture-characterization records (Lavrov, 2016; Kaukau, 2022), and partial or total losses alter return flow above the loss zone. Nevertheless, the critical equivalent-density limits $F_{crit,m}$ and model-disagreement intervals can be compared directly with any independently established formation limit at the same hydraulic state without repeating the friction calculations.

6. Conclusions

1. At fixed volumetric flow rate, the relative pressure-gradient truncation error of the linearized Bingham slot approximation is $E_P = \xi^3/2$, requiring $\xi \leq 0.464159$ ($Bi \leq 3.93620$) for a 5% tolerance. Because the nine Belhedan well sections operate at $\xi = 0.704$ –0.822, linearization overpredicts column annular friction by 6.97–9.24 psi (24.91–26.36%).
2. Across the 1,860-case dimensionless curvature survey ($0.20 \leq \kappa \leq 0.80$, $1 \leq Bi \leq 60$), the maximum sampled slot-to-annulus curvature error is 3.3589% (with 3.9496% at the separate Newtonian reference limit $Bi = 0$), and exact-slot losses in V33, V36, and V39 exceed concentric-annulus solutions by only 0.10–0.12 psi (0.34–0.37%). Independent annular quadrature reproduces the unrounded column totals and V39 sectional contributions.

3. Baseline annular friction consumes 55.22–68.44% of the downhole formation allowance (56.00–58.09 psi) while representing only 2.91–3.82% of standpipe pressure; consequently, hydraulic model adequacy for narrow-margin intervals must be evaluated against the local residual reserve R_a via the residual-margin ratio $\Lambda_m = \Delta P_m / R_a$.
4. At 1 vol% solids and $SG_s = 2.145$, the linearized-slot excess consumes 66.0%, 82.0%, and 162.1% of the concentric reserves in V33, V36, and V39; in V39, the margin changes from +5.78 to –3.59 psi (+5.44 psi with micromechanical coupling). At calcite-reference $SG_s = 2.71$, V33 and V36 remain positive under the concentric model (+1.28 and +0.69 psi) but turn negative under WellPlan (–1.75 and –2.83 psi) and the linearized slot (–5.79 and –7.37 psi), while V39 is negative across all models (–3.24 to –12.60 psi). At dolomite-reference $SG_s = 2.86$, all three wells are negative across all formulations.
5. Model-specific critical limits $F_{crit,m}$ define open disagreement intervals ($F_{crit,a} < F < F_{crit,m}$) that delimit where hydraulic-model choice changes the calculated margin sign against any independent formation threshold without repeating friction calculations. Separately, drill-pipe CCI values of 0.45–0.92 in the three wells indicate a need for closer hole-cleaning assessment rather than measured solids accumulation.

Declarations and Statements

Data Availability: The operating inputs, adopted formation-equivalent assessment limits, and WellPlan reference outputs used for the reported calculations are summarized in Table 1. Governing equations, dimensionless survey bounds, and numerical checkpoints are stated in the manuscript; the published loss-history data are cited to Kaukau (2022).

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Conflict of Interest: The authors declare that they have no conflicts of interest.

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