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Studying the State Feedback linearization and Input-Output State Feedback linearization by using Matlab Software to Notice the Systems Response for all Feedback Controls Applied

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المخلص:

المبادئ الأساسية لهذه الورقة حول دراسة التغذية الراجعة والتي فيها نهج شائع الاستخدام و يستخدم في التحكم في الأنظمة غير الخطية يتضمن النهج الخروج بتحويل النظام غير الخطي إلى نظام خطي مكافئ من خلال تغيير المتغيرات ومدخل تحكم مناسب. و تقتصر تقنيات الإدخال والمخرجات الخطية على العمليات التي تكون فيها هذه الديناميكيات الصفرية مستقرة ويتم تحقيق هذا الهدف من خلال اشتقاق مخرجات اصطناعية تنتج نموذجًا خطيًا للتغذية الراجعة بأبعاد الحالة ثم يتم تصنيع وحدة تحكم خطية للمدخلات الخطية نموذج الدالة وسيتم استخدام ثلاث حالات مختلفة للتحكم في التغذية الراجعة لتحقيق الاستقرار في النظام غير الخطي .

Abstract

The basic principles of this paper about the Feedback linearization is a common approach used in controlling nonlinear systems. The approach involves coming up with a transformation of the nonlinear system into an equivalent linear system through a change of variables and a suitable control input. Moreover, Input-output linearization techniques are restricted to processes in which these so-called zero dynamics are stable and this objective is achieved by deriving artificial outputs that yield a feedback linearized model with state dimension and a linear controller is then synthesized for the linear input-state model. Furthermore, in this paper three different cases of feedback control will be used to stabilize the nonlinear system.

Keywords*: State feedback linearization (SFL) . Input-output state feedback linearization (IOSF). Feedback (Fb). Input-State Linearization (ISL). Output-State Linearization (OSL)*

Introduction

This paper introduces the idea of state feedback linearization (SFL) and Input-output state feedback linearization (IOSF), and their application on several nonlinear systems. In addition, a simulation for all these problems is done by using Matlab Software to notice the systems response for all feedback controls applied. Moreover, in this paper three different cases of feedback control will be used to stabilize the nonlinear system and Feedback Linearization is the idea of feedback linearization is to introduce some transformation (usually to the system input) that makes the system between new input and output linear, and thus any linear control design is made possible.

The Equations used to Solve Problems:.

We will using the system for three different cases of feedback control will be used to stabilize the nonlinear system given by:

$\dot{x} = \bar{f}(x) + \bar{g}(x)\text{u} \qquad X \in R^n \quad \& \quad \text{u} \in R$

$\bar{f}\ ;\ R^n \Longrightarrow R^n \qquad \& \qquad \bar{g}\ ;\ R^n \Longrightarrow R^n$

where $\chi \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^p$ is the vector of inputs, and $\bar{f}, \bar{g} \in \mathbb{R}^n$ are the vector of outputs.

Input-State Linearization:

In general to know if the system is input state Linearization, and there must be a transformation ***T(x)*** that transform the system to linear model. Moreover, the transformation ***T(x)*** must be diffeomorphism and witch mean **T(x)** is continuous and $\mathbf{T^{-1}}(\mathbf{x})$ is exist and it is also continuous. The system that is represented by the oscillation equations.

The state Equation For First System:

$$x'_1 = -x_1 + x_2$$
$$x'_2 = x_1 - x_2 - x_1x_3 + u$$
$$x'_3 = x_1 + x_1x_2 - 2x_3$$
$$F(x) = \begin{bmatrix} -x_1 + x_2 \\ x_1 - x_2 - x_1x_3 \\ x_1 + x_1x_2 - 2x_3 \end{bmatrix}, \qquad \bar{g}(x) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

In the first we need get $\boldsymbol{Z = T(x)}$ and the transformation ***T(x)*** must satisfies these conditions

$$\frac{\partial T_1}{\partial x}\bar{g} = 0\,, \qquad \frac{\partial T_2}{\partial x}\bar{g} = 0, \qquad \frac{\partial T_3}{\partial x}\bar{g} \neq 0, \quad T_2(x) = \frac{\partial T_1}{\partial x}\bar{f}, \quad T_3(x) = \frac{\partial T_2}{\partial x}\bar{f}$$

$$\frac{\partial T_1}{\partial x}\bar{g} = 0 \Longrightarrow \begin{bmatrix} \frac{\partial T_1}{\partial x_1} & \frac{\partial T_1}{\partial x_2} & \frac{\partial T_1}{\partial x_3} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \frac{\partial T_1}{\partial x_2} = 0 \Longrightarrow T_1 = T_1(x_1, x_3) \qquad \frac{\partial T_2}{\partial x}\bar{g} = 0 \Longrightarrow$$
$$\begin{bmatrix} \frac{\partial T_2}{\partial x_1} & \frac{\partial T_2}{\partial x_2} & \frac{\partial T_2}{\partial x_3} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \frac{\partial T_2}{\partial x_2} = 0 \Longrightarrow T_2 = T_1(x_1, x_3)$$

$$\frac{\partial T_3}{\partial x}\bar{g} \neq 0 \Rightarrow \begin{bmatrix}\frac{\partial T_3}{\partial x_1} & \frac{\partial T_3}{\partial x_2} & \frac{\partial T_3}{\partial x_3}\end{bmatrix}\begin{bmatrix}0\\1\\0\end{bmatrix} = \frac{\partial T_3}{\partial x_2} \neq 0$$

$$T_2(x) = \frac{\partial T_1}{\partial x}\bar{f} = \begin{bmatrix}\frac{\partial T_1}{\partial x_1} & \frac{\partial T_1}{\partial x_2} & \frac{\partial T_1}{\partial x_3}\end{bmatrix}\begin{bmatrix}-x_1+x_2\\x_1-x_2-x_1x_3\\x_1+x_1x_2-2x_3\end{bmatrix}$$

$$\because \frac{\partial T_1}{\partial x_2} = 0 \Rightarrow \therefore\ T_2(x) = \frac{\partial T_1}{\partial x_1}(-x_1+x_2) + \frac{\partial T_1}{\partial x_3}(x_1+x_1x_2-2x_3)$$

$$T_3(x) = \frac{\partial T_2}{\partial x}\bar{f} = \begin{bmatrix}\frac{\partial T_2}{\partial x_1} & \frac{\partial T_2}{\partial x_2} & \frac{\partial T_2}{\partial x_3}\end{bmatrix}\begin{bmatrix}-x_1+x_2\\x_1-x_2-x_1x_3\\x_1+x_1x_2-2x_3\end{bmatrix}$$

$$\frac{\partial T_2}{\partial x_2} = 0 \Rightarrow\therefore\ T_3(x) = \frac{\partial T_2}{\partial x_1}(-x_1+x_2) + \frac{\partial T_2}{\partial x_3}(x_1+x_1x_2-2x_3)$$

Let's try $T_1 = x_1 + x_3 \Rightarrow T_2 = \frac{\partial T_1}{\partial x_1}(-x_1+x_2) + \frac{\partial T_1}{\partial x_3}(x_1+x_1x_2-2x_3)$

$$T_1 = (-x_1+x_2) + (x_1+x_1x_2-2x_3) = x_2 + x_1x_2 - 2x_3$$

We have found that it doesn't meet the condition.

Let's try $T_1 = x_1^2 + x_3 \Rightarrow T_2 = 2x_1(-x_1+x_2) + (x_1+x_1x_2-2x_3)$

$$T_1 = -2x_1^2 + 2x_1x_2 + x_1 + x_1x_2 - 2x_3 \Rightarrow T_2 = -2x_1^2 + 3x_1x_2 + x_1 - 2x_3$$

We have found that it doesn't meet the condition.

Let's try $T_1 = x_1^2 - x_3 \Rightarrow T_2 = 2x_1(-x_1+x_2) - (x_1+x_1x_2-2x_3)$

$$T_1 = -2x_1^2 + 2x_1x_2 - x_1 - x_1x_2 + 2x_3 \Rightarrow T_2 = -2x_1^2 + x_1x_2 - x_1 + 2x_3$$

And, we have found that it doesn't meet the condition because $T_1 = T_1(x_1, x_3)$

Let's try $T_1 = x_1^2 - 3x_3 \Rightarrow T_2 = 2x_1(-x_1+x_2) - 3(x_1+x_1x_2-2x_3)$

$$T_1 = -2x_1^2 + 2x_1x_2 - 3x_1 - 3x_1x_2 + 6x_3 \Rightarrow T_2 = -2x_1^2 - x_1x_2 - 3x_1 + 6x_3$$

Also, we have found that it doesn't meet the condition because $T_1 = T_1(x_1, x_3)$

Let's try $T_1 = x_1^2 + 2x_3^2 \Rightarrow T_2 = 2x_1(-x_1+x_2) + 4(x_1+x_1x_2-2x_3)$ $T_1 = -2x_1^2 + 2x_1x_2 + 4x_1 + 4x_1x_2 - 8x_3 \Rightarrow T_2 = 2x_1 + 4x_1x_2 - 8x_3$

It doesn't meet the condition because $T_1 = T_1(x_1, x_3)$

$T_1 = x_1^2 - 4x_3^2 \Rightarrow T_2 = 2x_1(-x_1+x_2) - 8(x_1+x_1x_2-2x_3)$

Let's try with

$T_1 = -2x_1^2 + 2x_1x_2 - 8x_1 - 8x_1x_2 - 16x_3 \Rightarrow T_2 = -2x_1^2 - 6x_1x_2 - 8x_1 - 16x_3$

Also, it doesn't meet the condition because $T_1 = T_1(x_1, x_3)$

Let's try to $T_1 = 5x_1^2 - 10x_3 \Rightarrow T_2 = 10x_1(-x_1+x_2) - 10(x_1+x_1x_2-2x_3)$

$$T_1 = -10x_1^2 + 10x_1x_2 - 10x_1 - 10x_1x_2 + 20x_3 \Rightarrow T_2 = -10x_1^2 - 10x_1 + 20x_3$$

We note that the last equation meets the condition:

$$T_3 = \frac{\partial T_2}{\partial x_1}(-x_1+x_2) + \frac{\partial T_2}{\partial x_3}(x_1+x_1x_2-2x_3)$$

$$T_3 = -20x_1 - 10\,(-x_1+x_2) + 20(x_1+x_1x_2-2x_3)$$

$$T_3 = 20x_1^2 - 20x_1x_2 + 10x_1 - 10x_2 + 20x_1 + 20x_1x_2 + -40x_3$$

$T_3 = 20x_1^2 + 30x_1 - 10x_2 - 40x_3$

$$T(x) = \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} = \begin{bmatrix} 5x_1^2 - 10x_3 \\ -10x_1^2 - 10x_1 + 20x_3 \\ 20x_1^2 + 30x_1 - 10x_2 - 40x_3 \end{bmatrix}$$

Now we have to find the $x = T^{-1}(z)$

$5\,x_1^2 - 10x_3 = Z_1 \Rightarrow 5x_1^2 = Z_1 + 10x_3 \Rightarrow Divisible\ by\ 5$

$x_1^2 = 0.2\,Z_1 + 2x_3$

$-10x_1^2 + 20x_3 - 10x_1 = Z_2 \Rightarrow Z_2 = -10\,(0.2\,Z_1 + 2x_3) - 10x_1 + 20x_3$

$\Rightarrow 10x_1 = -2Z_1 - Z_2 \quad Divisible\ by\ 10 \quad \Rightarrow x_1 = -0.2Z_1 - 0.1Z_2$

$x_1^2 = 0.2\,Z_1 + 2x_3 \quad \Rightarrow \quad 2x_3 = x_1^2 - 0.2\,Z_1$

$(-0.2Z_1 - 0.1Z_2)^2 - 0.2Z_1 = 2x_3 \Rightarrow x_3 = 0.02\,Z_1^2 - 0.1Z_1 + 0.02Z_1Z_2 + 0.005Z_2^2$

$Z_3 = 20x_1^2 + 30x_1 - 10x_2 - 40x_3$

$10x_2 = 20x_1^2 + 30x_1 - 40x_3 \quad - Z_3 \quad Divisible\ by\ 10 \Rightarrow \quad x_2 = 2x_1^2 + 3x_1 - 4x_3 \quad - 0.1Z_3$

$x_2 = 2(-0.2Z_1 - 0.1Z_2)^2 + 3(-0.2Z_1 - 0.1Z_2) - 4(0.02Z_1^2 - 0.1Z_1 + 0.02Z_1Z_2 + 0.005Z_2^2) - 0.1Z_3$

$\Rightarrow x_2 = -0.6Z_1 - 0.3Z_2 - 0.1Z_3 + 0.4Z_1 \Rightarrow x_2 = -0.2Z_1 - 0.3Z_2 - 0.1Z_3$

We can find $x = T^{-1}(z)$ in system matrix:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = T^{-1}(z) = \begin{bmatrix} -0.2Z_1 - 0.1Z_2 \\ -0.2Z_1 - 0.3Z_2 - 0.1Z_3 \\ 0.02Z_1^2 - 0.1Z_1 + 0.02Z_1Z_2 + 0.005Z_2^2 \end{bmatrix}$$

$\dot{Z_1} = 10x_1\dot{x_1} - 10\dot{x_3}$

$\dot{Z_1} = 10x_1(-x_1 + x_2) - 10(x_1 + x_1x_2 - 2x_3) = -10x_1^2 - 10x_1 + 20x_3$
$= -10(-0.2Z_1 - 0.1Z_2)^2 - 10(-0.2Z_1 - 0.1Z_2) + 20(0.02Z_1^2 - 0.1Z_1 + 0.02Z_1Z_2 + 0.005Z_2^2)$

$\dot{Z_1} = -0.4Z_1^2 - 0.4Z_1Z_2 - 0.1Z_2^2 + 2Z_1 + Z_2 + 0.4Z_1^2 - 2Z_1 + 0.4Z_1Z_2 + 0.1Z_2^2$

$\dot{Z_1} = Z_2$

$\dot{Z_2} = -20x_1\dot{x_1} - 10\dot{x_1} + 20\dot{x_3}$

$\dot{Z_2} = -20x_1(-x_1 + x_2) - 10(-x_1 + x_2) + 20(x_1 + x_1x_2 - 2x_3)$

$\dot{Z_2} = 20x_1^2 - 30x_1 - 10x_2 - 40x_3$

$\dot{Z_2} = 20(-0.2Z_1 - 0.1Z_2)^2 - 30(-0.2Z_1 - 0.1Z_2) - 10(-0.2Z_1 - 0.3Z_2 - 0.1Z_3) - 40(0.02Z_1^2 - 0.1Z_1 + 0.02Z_1Z_2 + 0.005Z_2^2) - 0.1Z_3$

$\dot{Z_2} = 0.8Z_1^2 + 0.8Z_1Z_2 + 0.2Z_2^2 - 6Z_1 - 3Z_2 + 2Z_1 + 3Z_2 + Z_3 - 0.8Z_1^2 + 4Z_1 - 0.8\,Z_1Z_2 - 0.2Z_2^2$

$\dot{Z_2} = Z_3$

$\dot{Z_3} = 40x_1\dot{x_1} + 30\dot{x_1} - 10\dot{x_2} - 40\dot{x_3}$

$\dot{Z_3} = 40x_1(-x_1 + x_2) + 30(-x_1 + x_2) - 10(x_1 - x_2 - x_1x_3 + u) - 40(x_1 + x_1x_2 - 2x_3)$

$\dot{Z_3} = -40x_1^2 - 80x_1 + 40x_2 + 10x_1x_3 + 80x_3 - 10u$

$$\dot{Z_3} = -40(-0.2Z_1 - 0.1Z_2)^2 - 80(-0.2Z_1 - 0.1Z_2) + 40(-0.2Z_1 - 0.3Z_2 - 0.1Z_3) + 10(-0.2Z_1 - 0.1Z_2)(0.02Z_1^2 - 0.1Z_1 + 0.02Z_1Z_2 + 0.005Z_2^2) + 80(0.02Z_1^2 - 0.1Z_1 + 0.02Z_1Z_2 + 0.005Z_2^2) - 10u$$

$$\dot{Z_3} = \underbrace{0.2Z_1^2 - 0.405Z_1^3 - 4Z_2 - 4Z_3 + 0.1Z_1Z_2 - 0.24Z_1^2Z_2 - 0.03Z_1Z_2^2}_{\boldsymbol{f(z)}} - \underbrace{10}_{\boldsymbol{g(z)}}u$$

Hence, deign SFL controller that generates $Z \to 0$ with the input and output.

$$u = \frac{1}{g(z)}[-f(z) + v]$$

And $\boldsymbol{v}$ is

$$v = k_1z_1 + k_2z_2 + k_3z_3$$
$$\boldsymbol{f(z)} = 0.2Z_1^2 - 0.405Z_1^3 - 4Z_2 - 4Z_3 + 0.1Z_1Z_2 - 0.24Z_1^2Z_2 - 0.03Z_1Z_2^2$$
$$\boldsymbol{g(z)} = 10$$

Since the k_1, k_2, k_3 should be chosen to place of the system poles in LHP

The system is state feedback linearizable and its global diffeomorphism , also we have to found many other points that can meet the condition in the first problem such that :

(1) $2x_1^2$ -$4x_3$ (2) $3x_1^2$ - $6x_3$ (3) x_1^2 - $2x_3$
(4) $4x_1^2$ - $8x_3$ (5) $6x_1^2$ - $12x_3$ (6) 7 x_1^2 - $14x_3$

Every those points can be meet the conditions $T_1 = T_1(x_1, x_3)$

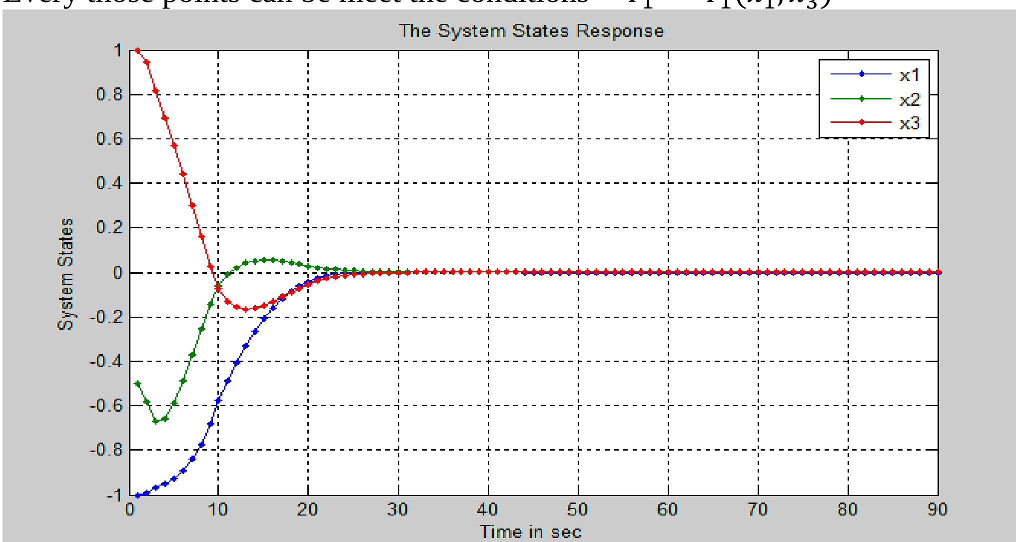

Figure (1) System States Response

The state Equation For First System:

And we want to Consider the system to make this's system input-output feedback linearizable:

$$x'_1 = -x_2 + 2x_1^2 \sin(x_1)$$
$$x'_2 = x_3 - u$$
$$x'_3 = -x_1 - x_3 \qquad y = x_1$$

By calculating the derivative of the output $\boldsymbol{y}$ we get:

$$T_2 = y = x_1 = h(x)$$
$$T_3 = \dot{y} = \dot{x_1} = x_2 + 2x_1^2 - \sin(x_1) = L_f h(x)$$
$$\ddot{y} = \dot{x_2} + 4x_1\dot{x_1} - \dot{x_1}\cos(x_1)$$

$$= x_3 - u + 4x_1(x_2 + 2x_1^2 - \sin(x_1)) - (x_2 + 2x_1^2 - \sin(x_1)\,cos(x_1)$$

$$\Longrightarrow \ddot{y} = x_3 - u + 4x_1x_2 + 8x_1^3 - 4\,x_1\sin(x_1) - x_2 co\,s(x_1) - 2x_1^2 co\,s(x_1) + \sin(x_1)\,cos(x_1)$$

Also, the system has a relative degree 2 in R^3 and is input-output linearizable in D

To find $\boldsymbol{T_1(x)}$

$$\frac{\partial T_1}{\partial x}\bar{g} = \begin{bmatrix}\frac{\partial T_1}{\partial x_1} & \frac{\partial T_1}{\partial x_2} & \frac{\partial T_1}{\partial x_3}\end{bmatrix}\begin{bmatrix}0\\-1\\0\end{bmatrix} = \frac{\partial T_1}{\partial x_2} = 0 \implies T_1 = T_1(x_1, x_3)$$

Let's choose $T_1 = x_3$ because it is satisfies $\frac{\partial T_1}{\partial x}\bar{g} = 0$

$$\begin{bmatrix}q_1\\Z_1\\Z_2\end{bmatrix} = \boldsymbol{T_1(x)} = \begin{bmatrix}x_3\\x_1\\x_2 + 2x_1^2 - \sin(x_1)\end{bmatrix} = \begin{bmatrix}T1_{(X)}\\T2_{(X)}\\T3_{(X)}\end{bmatrix}$$

Also, note that:

$$\begin{bmatrix}x1\\x2\\x3\end{bmatrix} = T^{-1}\binom{q}{x} = \begin{bmatrix}Z_1\\Z_2 - 2Z_1^2 + \sin(x_1)\\q_1\end{bmatrix}$$

<u>Verifying:</u>

$$\dot{q_1} = \dot{x_3} = -x_1 - x_3 = -Z_1 - q_1$$

$$\dot{Z_1} = \dot{x_1} = x_2 + 2x_1^2 - \sin(x_1)$$

$$= Z_2 - 2Z_1^2 + \sin(Z_1) + 2Z_1^2 - \sin(Z_1) = Z_2$$

$$\dot{Z_2} = \dot{x_2} + 4x_1\dot{x_1} - \dot{x_1}\cos(x_1)$$

$$\dot{Z_2} = q_1 - u + 4Z_1(Z_2 - 2Z_1^2 + \sin(Z_1) + 2Z_1^2 - \sin(Z_1)) - \cos(Z_1)(Z_2 - 2Z_1^2 + \sin(Z_1) + 2Z_1^2 - \sin(Z_1)$$

$$= q_1 - u + 4Z_1Z_2 - Z_2\cos(Z_1)$$

Then the linearizing feedback control law is : $u = [q_1 + 4Z_1Z_2 - K_1Z_1 - K_2Z_2]$

Focusing on zero dynamic and assuming $Z = 0$

In tracking case for the reference signal $(t) = \sin(t)$, now we will find the tracking Error:

Let $e = y - r(t) = x_1 - r(t) \Longrightarrow \dot{e} = \dot{x_1} - r(t) \Longrightarrow \ddot{e} = \ddot{y} - \ddot{r}$

$$e = x_3 - u + 4x_1x_2 + 8x_1^3 - 4\,x_1\sin(x_1) - x_2 co\,s(x_1) - 2x_1^2 co\,s(x_1) + \sin(x_1)\,co\,s(x_1) + \sin(t)$$

$$u = x_3 - u + 4x_1x_2 + 8x_1^3 - 4\,x_1\sin(x_1) - x_2 co\,s(x_1) - 2x_1^2 co\,s(x_1) + \sin(x_1)\,co\,s(x_1) + \sin(t) - K_1e - K_2\dot{e}$$

By using MATLAB software to calculate the co*ntrol* input u and applying it to the system model, we get the response as shown in figures (2), (3) and(4) . As we will that figure (2) show that the output y asymptotically tracks the reference signal $r(t) = \sin(t)$ and the figure (3) show the system state versus time x, and figure (4) show the Error dynamics versus time.

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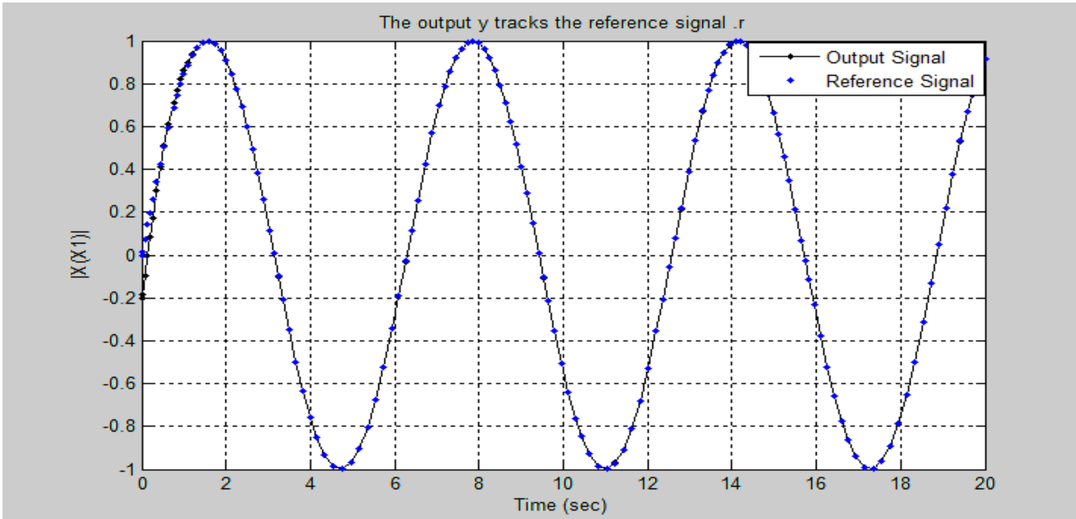

Figure (2) The Output Y tracks The Reference Signal r = sin (t)

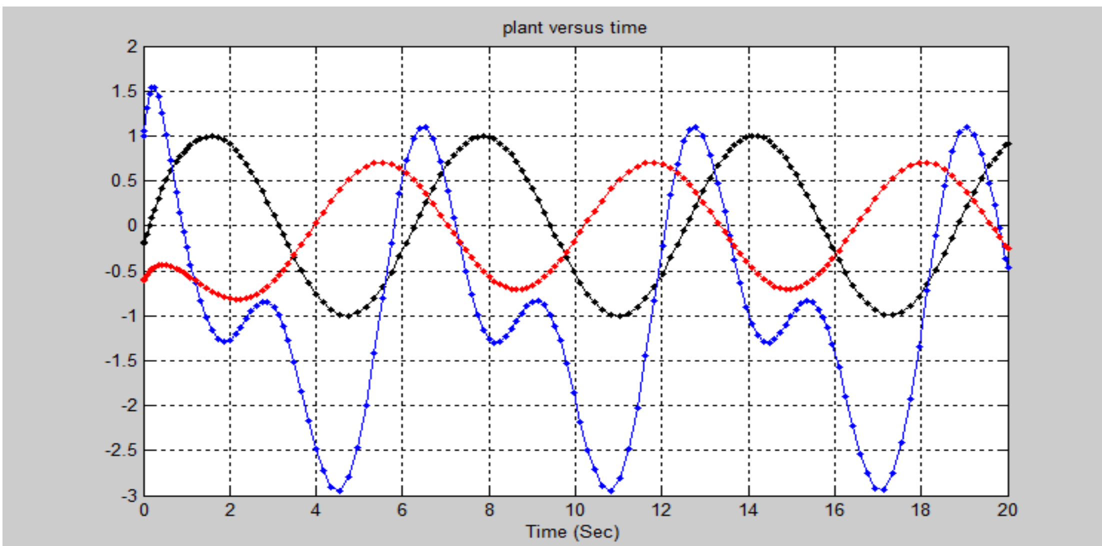

Figure (3) the system states verses time

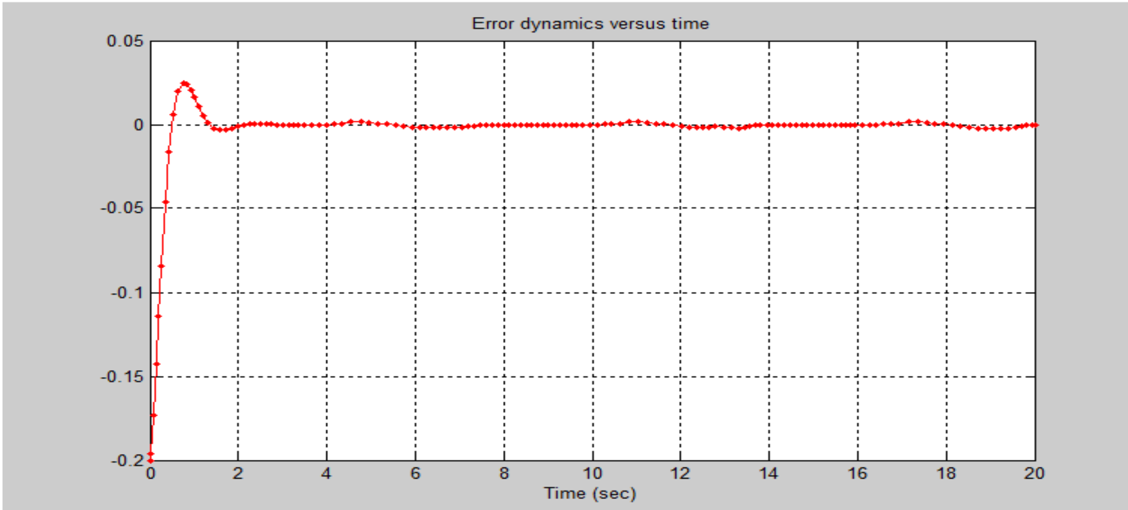

Figure (4) show the Error Dynamics verses time

Ball and Beam Experiment:

Here, we will simulate the ball and beam experiment given of the ball and beam handout on the web site and repeated here for convenience:

let a $=\frac{mr2}{l2}$, b = mg , c = $\frac{r}{l}$,

d= $\frac{Jb}{r2}$ + m , e = $K_m K_g$, f = $\frac{RmJeg}{KmKg}$, where $J_b = \frac{2}{5} mr^2$ is the moment of inertia of the ball. Moreover, let $z_1 = x$, $z_2 = \dot{x}$, $z_3 = \theta$, $z_4 = \dot{\theta}$, the ball and beam system can be represented as

$\dot{z}_1 = z_2$

$\dot{z}_2 = \frac{a}{d} z_1 z_4^2 - \frac{b}{d} \sin(cz3)$

$\dot{z}_3 = z_4$

$\dot{z}_4 = \frac{Vin}{f} - \frac{e}{f} z_4$

Where the input is V_{in} and the output equation is given by → y = x = z_1

For the simulation, we will use the following system parameters:

- Motor torque constant, k_m= 0.00767 *N . m / amp*
- Armature resistance, R_m = 2.6 *ohm*
- Equivalent moment of inertia, J_{eq} = 0.0029 $kg.\ m^2$
- Gear ratio, k_g= 70
- Radius of the gear, r = 2 *cm*
- Length of the beam, l = 34 *cm*
- Gravitational force constant, g = 9.8 $^{m}/_{s2}$
- Mass of the ball, m = 0.68 *kg*

In this system we want to show that it's not input-output feedback linearizable and why is this case. Although we cannot apply the standard feedback linearizable technique to this system and we will use approximate feedback linearization instead. Note that term $\frac{2a}{df} z_1 z_4 V_{in}$ from $y^{(3)}$ can be expected to be small in magnitude when the state is close to

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the equilibrium. Furthermore, we will simply ignore this term and proceed and we can show that by doing this the system has relative degree 4. In addition, we will design a feedback linearizing controller based on the approximate feedback linearization idea and simulate it on the plant and show via plots that the controller regulates the position of the ball to zero.

The output should be differentiated until the input appear:

$$y = z_1$$
$$\dot{y} = \dot{Z}_1 = Z_2$$
$$\ddot{y} = \dot{Z}_2 = \frac{a}{d} Z_1 Z_4^2 - \frac{a}{d}\sin(cZ_3)$$
$$\dddot{y} = \frac{a}{d} \dot{Z}_1 \dot{Z}_4 + 2\frac{a}{d} Z_1 Z_4 \dot{Z}_4 - \frac{b}{d} cZ_4 \cos(cZ_3)$$
$$\dddot{y} = \frac{a}{d} Z_2 Z_4^2 + 2\frac{a}{d} Z_1 Z_4 \left(\frac{V_{in}}{f} - \frac{e}{f} Z_4\right) - \frac{b}{d} cZ_4 \cos(cZ_3)$$
$$\Longrightarrow \dddot{y} = \frac{a}{d} Z_2 Z_4^2 - 2\frac{a}{d}\frac{e}{f} Z_1 Z_4^2 - \frac{b}{d} cZ_4 \cos(cZ_3) + \frac{2a}{df} Z_1 Z_4 V_{in}$$
$$\because u = \frac{1}{g(z)}[-f(z) + v]$$

Where

$$f(z) = \frac{a}{d} Z_2 Z_4^2 - 2\frac{a}{d}\frac{e}{f} Z_1 Z_4^2 - \frac{b}{d} cZ_4 \cos(cZ_3)$$
$$g(z) = \frac{2a}{df} Z_1 Z_4$$

The control coefficient $g(z)$ is zero whether the beam angular velocity Z_4 or ball position Z_1 are going to zero. And, the exact input output feedback linearization approach is not applicable for this system. Also, at this part the beam angular velocity $z_{4=\dot{\theta}}$ is zero as well see the ball position from the center of the beam $z_1 = x$, and the method of input-output feedback linearizable IOFL is not practical for this system.

We cannot apply the standard feedback linearization IOFL technique to this system, so we will use approximate feedback linearization technique is to drop the $\frac{2a}{df} Z_1 Z_4 V_{in}$ from $y^{(3)}$ can expected to be small in magnitude when the state is close to the Equilibrium. We will simply ignore this term and then derivative the output y until we get u. Therefore, this system has relative degree 4 and design a feedback linearizing controller based on the approximate feedback linearization show via plots that the controller regulates the position of the ball to zero.

$$\dddot{y} = \frac{a}{d} Z_2 Z_4^2 - 2\frac{a}{d}\frac{e}{f} Z_1 Z_4^2 - \frac{b}{d} cZ_4 \cos(cZ_3) + \frac{2a}{df} Z_1 Z_4 V_{in}$$
$$y^{(4)} = \frac{a}{d} \dot{Z}_2 \dot{Z}_4 + 2\frac{a}{d} Z_4 \dot{Z}_4 - \frac{2ae}{df} \dot{Z}_1 \dot{Z}_4 - \frac{4ae}{df} Z_1 Z_4 \dot{Z}_4 - \frac{b}{d} c\dot{Z}_4 \cos(cZ_3) + \frac{b}{d} c^2 Z_4^2 \sin(cZ_3)$$

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$$y^{(4)} = \frac{a^2}{d^2} Z_1 Z_4^4 - 4\frac{ae}{df} Z_2 Z_4^4 + 4\frac{ae^2}{df^2} Z_1 Z_4^2 - \frac{ab}{d^2} Z_4^2 \sin(cZ_3) + \frac{b}{d} c^2 Z_4 \sin(cZ_3) + \frac{be}{df} cZ_4 \cos(cZ_3) + \frac{2a}{df} Z_2 Z_4 - \frac{4ae}{df^2} Z_1 Z_4 - \frac{bc}{df} \cos(cZ_3) V_{in}$$

$Z_1 = x_1$
$\dot{x_1} = Z_2 = x_2$
$\dot{x_2} = Z_3$
$\dot{x_3} = Z_4$
$\dot{x_4} = y^{(4)}$

Here, we will simply show that by doing this the system has relative degree 4.

$u = \frac{1}{g(z)}[-f(z) + v]$ We have to find that:

$$f(z) = \frac{a^2}{d^2} Z_1 Z_4^4 - 4\frac{ae}{df} Z_2 Z_4^4 + 4\frac{ae^2}{df^2} Z_1 Z_4^2 - \frac{ab}{d^2} Z_4^2 \sin(cZ_3) + \frac{b}{d} c^2 Z_4 \sin(cZ_3) + \frac{be}{df} cZ_4 \cos(cZ_3) + \frac{2a}{df} Z_2 Z_4 - \frac{4ae}{df^2} Z_1 Z_4$$

$$g(z) = \frac{bc}{df} \cos(cZ_3)$$

Then

$u = v = -K_1 y - K_2 \dot{y} - K_3 \ddot{y} - K_4 \dddot{y}$

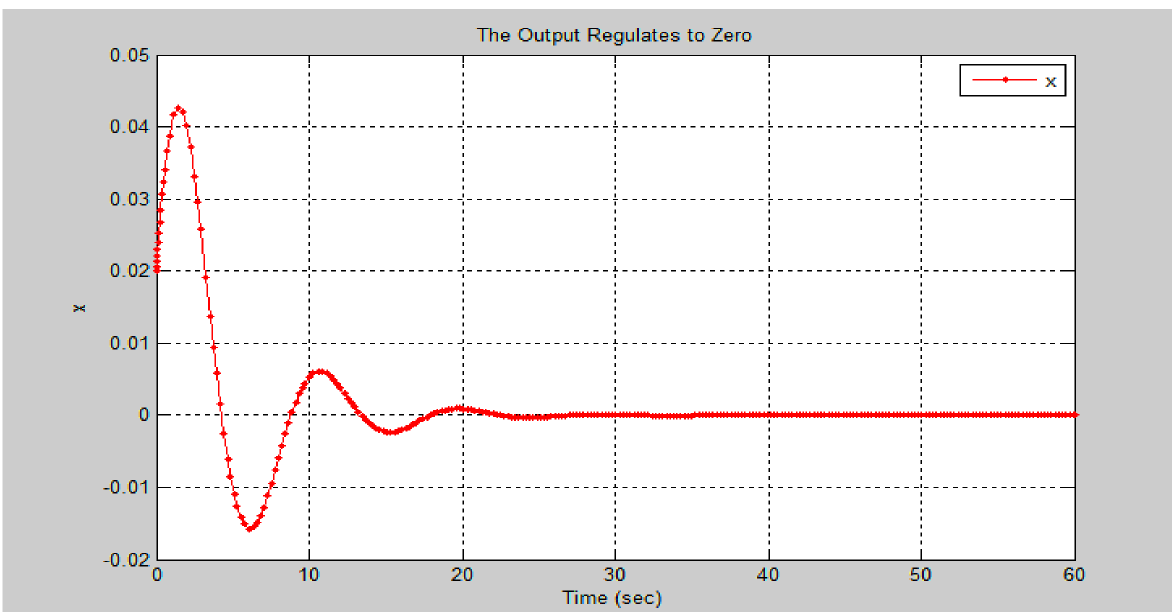

Figure (5) show the position of the ball regulates to zero

Conclusion:

According to the results we got from this paper that the first system was examined to check if it's a state feedback linearizable or not. Then a state feedback control law was designed to stabilize the origin to zero from non-zero initial condition. Moreover, we have found the system is a state feedback linearzable and globally diffeomorphism. The second system was tested to know if it's possible to apply an input-output feedback linearization. Then a state feedback control law was designed such that the output y asymptotically tracks the reference signal r(t)=sin(t).

Finally in the third system a simulation of a Ball beam experiment was implemented also the experiment was checked if it is Input-output state feedback linearizable or not. And how can we apply a feedback approximate linearization instead of IOSF because IOSF cannot be applied on this experiment.

References:

1- A. K. Adebekun and F. J. Schork. On the global stabilization of *n-th* order reactions. *Chem. Eng. Commun., 100:47-59, 1991*.
2- D. K. Adebekun and F. J. Schork. Continuous solution polymerization reactor control. 1. Nonlinear reference control of methyl methacrylate polymerization. *Ind. Eng. Chem. Res., 28:1308-1324, 1989.*
3- P. Agrawal, G. Koshy, and M. Ramseier. An algorithrn for operating a fed-batch reactor at optimum specific growth rate. *Biotech. Bioeng,. 33:115, 1989*
4- K. Adebekun. The robust global stabilization of a stirred tank reactor. *AICH J.38:65, 1992*
5- A.W. Alosp and T. F. Edgar. Nonlinear heat exchanger control *Commun., 75:155-17, 1990*
6- P. Brunovsky. A classification of linear controllable systems. Kybernetica. 19 (6):173–187.
7- 2.A. J. Krener. On the equivalence of control systems and the linearization of nonlinear systems. SIAM Journal on Control. 1973;11:670–676.
8- 3.R. W. Brockett. Feedback invariants for nonlinear systems. Proceedings of 7th IFAC Congress, *Helsinki. 1978;:1115–1120.*
9- 4.B. Jakubczyk and W. Respondek. On linearization of control systems. Bulletin Academie Polonaise des Sciences Series Mathematics. *1980;28:517–522.*
10- 5.L. R. Hunt and R. Su. Linear equivalents of nonlinear time varying systems. In: Proceedings of Mathematical Theory of Networks & Systems; August 5-7; Santa Monica, *CA. USA:1981. p. 119–123.*
11- 6.B. Charlet, J. Levine and R. Marino. Dynamic feedback linearization, SIAM Journal on *Control Optimization. 1991;(29):38–57.*
12- 7.B. Charlet, J. Levine and R. Marino. Sufficient conditions for dynamic state feedback linearization. Systems & Control Letters. *1989;(13):143–151.*
13- 8.Zhendong Sun and S. S. Ge, Nonregular feedback linearization: a nonsmooth approach, in IEEE Transactions on Automatic Control, *vol. 48, no. 10, pp. 1772–1776, Oct. 2003.*
14- 9.S-J. Lie and W. Respondek. Orbital feedback linearization of multi-input control systems. International Journal of Robust and Nonlinear Control. *2015;25(1):1352–1378*