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Adaptive and identification two tanks system in control

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Abstract

The classical approach to system identification is to identify the transfer function on the basis of measured data. The goal of this research, with the aid of Matlab package, is to both identify and adaptively control a laboratory non-linear two tanks system which is simulated in a Simulink model block. Sampling time and model structure selection will be introduced first. Then, concentration will be on the OLS and IV methods of estimating parameters as they will be used in the Matlab identification toolbox. The system accordingly will be identified to obtain both discrete and continuous linear function models at operating points of 3 and 6. Adaptive and self-tuning control is the second part of this report where self-tuning PI control will be implemented based on RLS and phase margin design algorithms. Relay-based auto-tuning PI control is the second part control method, and practical comparison between them is provided at the end of this report.

Introduction

Automatic control applications usually need a compact and accurate description of the system dynamic behaviour. Systems can be described using dynamic models which can be constructed based on physical laws, chemistry and so forth. However, such models are often difficult to derive, complex, time-consuming in simulation and consequently not suitable for fast online applications. System identification is the alternative way where observed input and output data is the source of estimating the dynamic model of a system. The type of model structure to be used is of a great decision as a primary step which based on knowledge of the system under consideration. Input signal is also important which can be chosen so that the model captures the behaviour of the system. Input and output data can be extracted as an identification experiment to be used in the identification process. Then, estimating the parameters of the selected model structure is the next step. Finally, the obtained model can be evaluated to see how the model can fit the system. The method of adaptive and self-tuning control is to be implemented in this report to the two tanks system using special mechanisms to successfully approach good results. These steps will be discussed in a bit of detail in the next two parts, system identification and adaptive control.

System identification:

Sampling time:

Sampling time is one of the key decisions to be made when identifying a process or a control system. Natural tendency is to sample as fast as possible in order to identify the system accurately, but that will simply lead to load disturbance (noise) as the intervals between the samples are too short and consequently the noise will be modelled as well.
On the contrary, if the sampling interval is too long, full behaviour of the system will not be accurately identified because of the lost dynamics information between any two samples as in figure (1-1), where the significant change in the process between the samples (r-3) and (r-4) will not be taken into consideration during the process of identification.

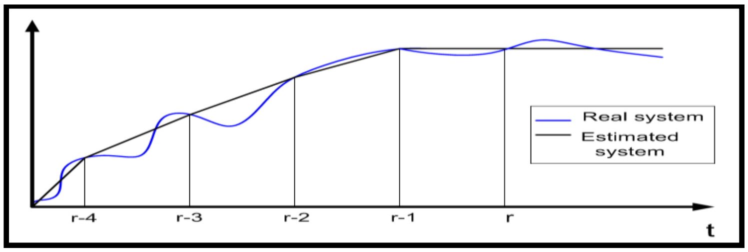

Figure (1-1) Estimating the real system.

As a result, an appropriate solution is the implementation of a good rule of thumb which based on observing the rise time (t_r) upon an initial step test which meets 95% of the steady state response. Figure (1-2) shows the step response of the two tanks system at an output operating point of 3 as an example where t_r is found to be 375sec.

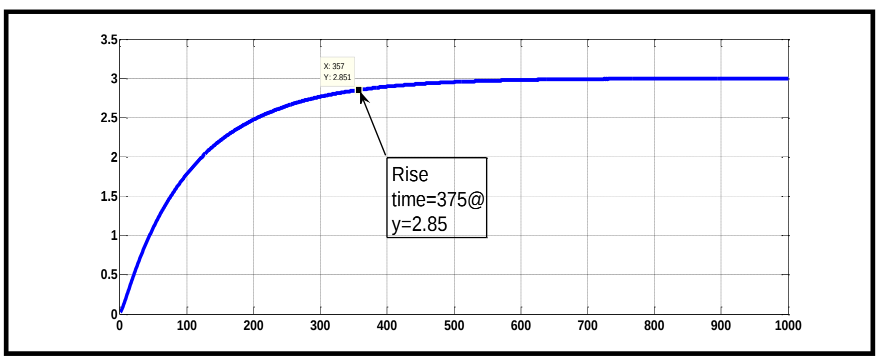

Figure (1-2) Step response of the two tanks system.

Then the rule of thumb can be applied which says that the sampling time (T_s) should be 6 to 10 times faster than the system rise time as follow:

$$\frac{t_r}{10} < T_s < \frac{t_r}{6} \Longrightarrow 37.5 < T_s < 62.5$$

Model structure:

It may take considerable amount of time among a very large number of model structures to be checked out, but it often takes far shorter time to compute and evaluate a model in a certain structure. This indicates the importance of selecting the model structure. This selection must be based on first understanding the identification procedure and second on the system to be identified. This combination would take us to the main aspects that influence the selection of the model structure like:

1- The type of model.
2- The model set which includes the model order.
3- Selecting a criterion to measure the closeness and compare the dynamic behaviour of both the model and the physical process.

It is now obvious that the model structure is the core of the identification process which would boil down the optimization algorithm run on the selected structure with the parameterization criterion. This difficulty is present especially in nonlinear systems, which is the case in the two tanks system, but once the proper structure has been selected; parameterization will be only soluble numerical problem.

ARMA (p, q) model is common in system identification which a way of forecasting model or process and representing both auto-regression analysis and moving average.
methods and taking the next structure:

$$\frac{Y_r(z)}{U_r(z)} = \frac{b_a+b_1z^{-1}+b_2z^{-2}+ \ldots\ldots\ldots\ldots+b_qz^{-q}}{1+a_1z^{-1}+a_2z^{-2}+ \ldots\ldots\ldots\ldots+a_pz^{-p}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (2\text{-}1)$$

Where (p) is the order of the autoregressive part, and (q) is the order of the moving average part. In our system (two tanks system); ARMA model would represent the system accurately as we will see.

First, we need to know the optimal number of parameters that gives the best fit. Young's information criterion denoted as (YIC) is one of the model structure selection methods which is represented in the next equation:

$$\text{YIC}= \ln(R) + \ln(S) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (2\text{-}2)$$

Where $R = \frac{\sigma_e^2}{\sigma_y^2} = \frac{\frac{1}{r}\sum_{i=1}^{r}[(y_i-\hat{y}_i)-e_{mean}]^2}{\frac{1}{r}\sum_{i=1}^{r}(\hat{y}_i-y_{mean})^2}$

And σ_e^2 is the error variance.

σ_y^2 is the output variance.

$$S = \frac{1}{n}\sum_{i=1}^{n}\frac{\sigma_i^2}{\hat{\theta}_i^2}$$

Where σ_i^2 is the parameter variance.

And $\hat{\theta}_i^2$ is the parameter estimates.

Score is the alternative way of representing this information and is defined as follow:

Score= $50 - YIC$

The provided program YIC.m will perform this criterion in parallel with the rlss.m program which is the recursive least square method (discussed later) of estimating ($\widehat{\Theta}$). The YIC program needs some variables to be carried out which are as follow:

$n_a = 1\ to\ 5 \Longrightarrow$ Number of denumerator coefficients.

$n_b = 1\ to\ 5 \Longrightarrow$ Number of numerator coefficients.

$n_k = 0\ to\ 6 \Longrightarrow$ First non-zero element in the numerator.

The given ranges cover typical industrial processes, and consequently will cover our system.

First, the input and output of the two tanks system is loaded to the workspace by running the Simulink model nltt2011, then the next command was executed that will result in the set of n_a n_b n_k , the score and the model fit respectively with the element of choice of 3.

>> [best_nn, best_score best_fit] = yic(y, u, 5, 5, 6)

Test results		
Model structure	**Score**	**Fit %**
na nb nk		
1 2 1	69.3837	99.9881
5 1 1	65.2487	99.9928
2 2 1	65.0733	99.9957

Table (1-1) Examining the model structure.

From the results, it can be noted that the best model structure for the two tanks system is (1 2 1) with a score of 69.3837 and a fit of 99.9881.

Non-linear two tanks system identification:

Sampling time:

Sampling time was chosen based on the rule of thumb discussed in a previous section and it lies within the range of:

$\frac{375}{10} < T_s < \frac{375}{6} = 37.5 < T_s < 62.5$ at an output operating point of 3.

$\frac{506}{10} < T_s < \frac{506}{6} = 50.6 < T_s < 84.3$ at an output operating point of 6.

The sampling time was chosen to be 40sec.

Input source:

Input signal to the system to be identified is of great importance. Step input is one of the possibilities, but this type of input seems to be impractical to use in identifying a system that have faster dynamics. PRBS is to be applied with the impulse input for correlation analysis because this combination signal changes sufficiently in order to excite the system so that the collected data will contain enough information about the dynamics of the system.

Model structure:

Two types of model structure are going to be used to identify the two tanks system, ARMA model representing the discrete-time model and continuous time FOPDT model.

Continuous time first order plus dead time model (FOPDT):
Many industrial processes can be adequately represented by FOPDT which takes the next form:

$$G(s) = \frac{Ke^{-sd}}{1+s\tau}$$

K is the gain, τ is the time constant and d is the transport delay.
This model will be used to identify the two tanks system.

Two tanks system identification procedure:

The Matlab package made it easy for a designer to identify a specific process using, for example, the techniques discussed previously. Identification toolbox constructs the mathematical model of dynamic systems according to the extracted data from the input and output through the workspace provided in the Matlab.

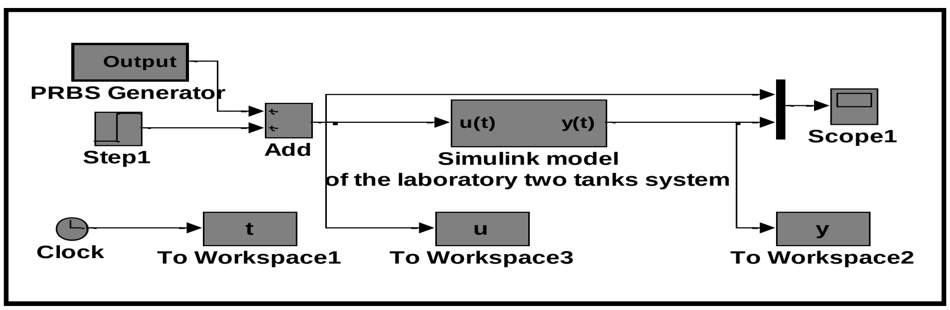

Figure (1-3) Simulink model of the two tanks system.

Using the simulink in figure (1-3), the output was observed to take the steady state part of the system response to be identified and omit the first 15 samples (the unsteady state response) and at the same time to couple the input with the output using the next set of commands:

```
>> u=u(15:end)
>> y=y(15:end)
>> t=t(15:end)
>> u=u-5.181
>> y=y-3
```

The resultant graph is shown in figure (1-4) where the green line is the two tanks system output part to be identified.

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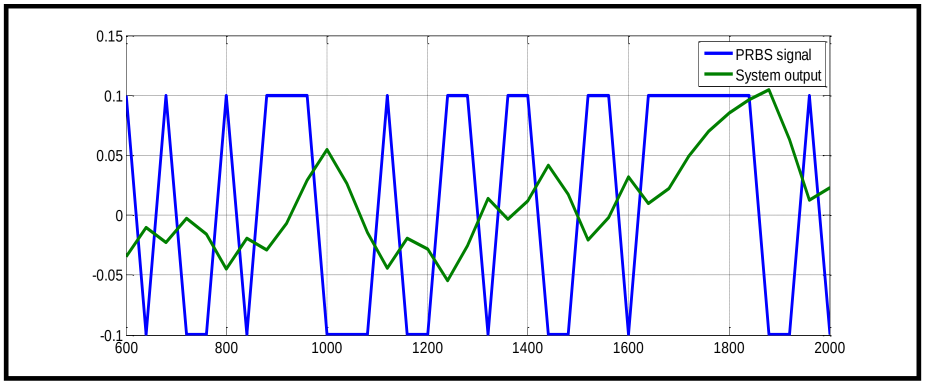

Figure (1-4) Output of the two tanks system coupled with the PRBS input signal.

The next figures summarize the main steps of identification using the ident toolbox for both ARMA model and FOPDT model.

(1)ARMA model:

The next command is executed to access the identification toolbox:

>> ident

Then the identification toolbox appears where the input and output of the system (u and y) are imported, estimation of the system starts with selecting the (linear parametric model) from the pop up list found in the estimate window. The model structure was chosen to be (1, 2, 1) that meets the best score. Two methods are available for estimating the ARMA model; least squares denoted (ARX) and instrumental variable denoted (IV) where two of them were examined and a better result was achieved using the ARX method with (98.89 %) fit to the system as shown in figure (1-5).

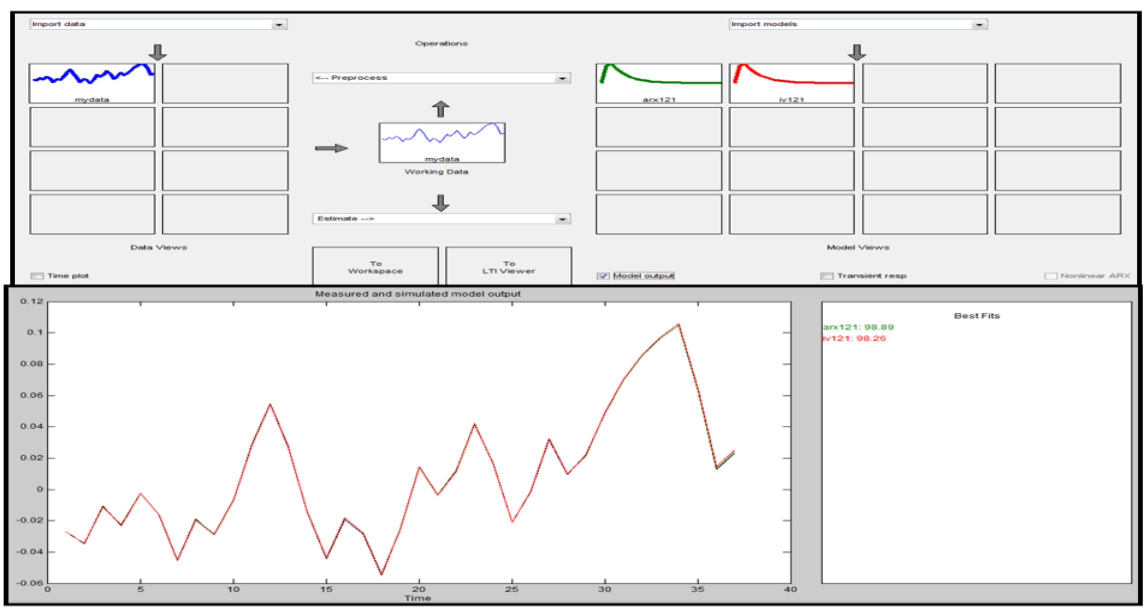

Figure (1-5) Results of estimating the two tanks system with LS and IV methods.

This identification was done at an operating point of 3 and the resultant z-transfer function is as follow:

$$G_3(z) = \frac{0.2388z^{-1} + 0.09573z^{-2}}{1 - 0.7396z^{-1}}$$

And the same procedure was done at an operating point of 6 using the structure (2, 2, 1) from table (1-2) and the result is as below:

$$G_6(z) = \frac{0.2111z^{-1} + 0.1007z^{-2}}{1 - 0.9273z^{-1} + 0.1016z^{-2}}$$

FOPDT model:

The same command and the same identification toolbox but here the process models (representing the continuous time domain) is selected from the estimate pop up window, then the simple FOPDT is selected which resulted in 97.96% fit to the system shown in the next figure (1-6).

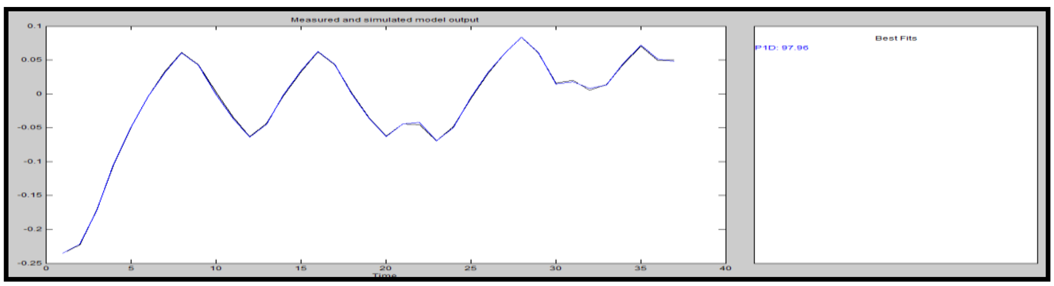

Figure (1-6) Result of estimating the two tanks system with continuous time FOPDT model.

The resultant transfer function at an operating output point of 3 is

$$G(s) = \frac{1.8216e^{-0.43504s}}{1 + 4.6827s}$$

And at an operating point of 6 the transfer function is:

$$G(s) = \frac{1.8077e^{-0.42573s}}{1 + 4.6263s}$$

These transfer functions were tested by comparison with two tanks system using the Simulink model in appendices A and B and the results were more than satisfied.

Adaptive and self-tuning control:

Adaptive control is the control technique used by a controller so to adapt to a controlled system. This adaptive controller figure (2-1) has a fixed structure with adjustable parameters and a certain mechanism to automatically adjust those parameters. Thus adaptive control theory is the process of finding parameter adjustment algorithms that guarantee global stability. Least squares algorithm is one of the mechanisms to estimate those parameters.

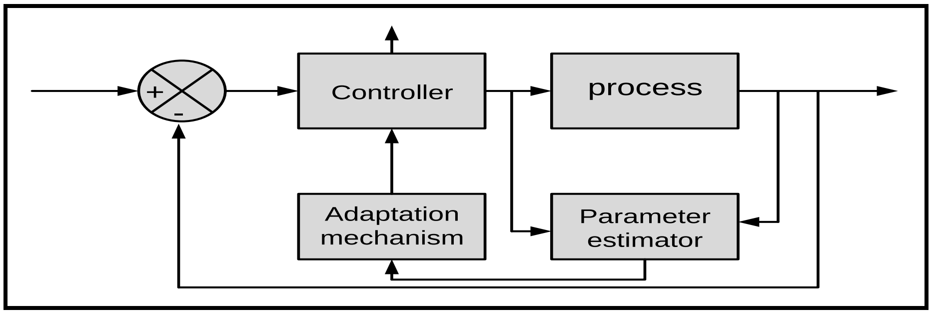

Figure (2-1) Adaptive control scheme.

Online adaptation mechanism:

After estimating the model parameters using the RLS, online adaptation of the controller is the next step where adaptive PI controller is to be used in controlling the two tanks system.

Self-tuning PI control and relay-based auto-tuning PI control with their applications on the two tanks system are going to be discussed in the next two sections.

Self-tuning PI control:

In many practical applications it is difficult to determine the controller parameters as the process dynamics and its disturbance are time-varying and unknown. Thus, process parameters have to be estimated. RLS, as it has been explained, is the method applied to estimate these parameters in this part of the experiment. It is thus desirable to use a controller that tunes its parameters online.

Discrete PI controller is the class to be implemented here, in parallel with the modified discrete FOPDT model that has 6 numerator coefficients which will model the two tanks system with a sampling time of T_s.

The modified z-transform FOPDT model is:

$$G(z) = \frac{b_\delta z^{-\delta} + b_{\delta+1} z^{-(\delta+)}}{1 - a_1 z^{-1}}$$

$$a_1 = e^{-\frac{T_s}{\tau}}$$

$$b_\delta = \frac{K}{\tau}\left[1 - e^{-(1-\alpha)\frac{T_s}{\tau}}\right] b_{\delta+1} = \frac{K}{\tau}\left[e^{-(1-\alpha)\frac{T_s}{\tau}} - e^{-\frac{T_s}{\tau}}\right]$$

And the integer and the fractional parts ($\delta\ and\ \alpha$) respectively are as follow:

$\delta = \left\lfloor \frac{d}{T_s} \right\rfloor + 1$ and $\alpha = \frac{d}{T_s} - \left\lfloor \frac{d}{T_s} \right\rfloor$

The continuous time transfer function of the PI controller is:

$$G_c(s) = K_p[1 + \frac{K_i}{S}]$$

Now if we specify the useful approximation of S into Z as follow:

$S \approx \frac{1 - z^{-1}}{T_s}$

We get: $G_c(z) = k_p[1 + \frac{T_s K_i}{1 - z^{-1}}$

And can be re-arranged as follow:

$G_c(z) = K_p(1 + T_sK_i)[\frac{1-\frac{1}{(1+T_sK_i)}z^{-1}}{1-z^{-1}}]$ .... (2-7)

K_i can be obtained from: $K_i = \frac{1-\frac{1}{a_1}}{T_s}$

After applying $K_i$and approximating equation (2-7) and multiplying the 6 coefficient FOPDT transfer function by $G_c(z)$ to obtain the open loop TF we get:

$G_{ol}(z) = \frac{K_p(1+T_sK_i)(b_1z^{-1}+b_2z^{-2}+b_3z^{-3}+b_4z^{-4}+b_5z^{-5}+b_6z^{-6})}{1-z^{-1}}$ .... (2-8)

The integral gain now is solved and K_p, the proportional gain, need to be set. Here, the value of K_p can be obtained from phase margin specifications, which is well known robustness parameter. The following two equations express the phase margin:

$|G_c(j\omega_g) * G(j\omega_g)| = 1$ .... (2-9)

$\phi_m = \pi + arg\{G_c(j\omega_p) * G(j\omega_p)\}$ .... (2-10)

Where ϕ_m is the phase margin, and the recommended range of ϕ_m lies in the next range ($30^0 < \phi_m < 60^0$).

ω_p is the phase crossover frequency, and based on the sampling theorem it satisfies the next range($0 \le \omega_p \le \frac{\pi}{T_s}$) and verifies the equation (2-10).

Once ω_p is obtained, ϕ_m has to be defined by selecting K_p so that;

$|G_{ol}(z)| = 1$ at $\omega = \omega_p$

And the amplitude of the equation (2-8) can be expressed as follow:

$$|G_{ol}(z)| = K_p(1 + T_sK_i)\left[\frac{\sqrt{\left[\sum_{i=1}^{i=6} b_i\, cosi\omega_{gc}T_s\right]^2 + \left[\sum_{i=1}^{i=6} b_i\, sini\omega_{gc}T_s\right]^2}}{\sqrt{\left[1 - cos\omega_{gc}T_s\right]^2 + \left[sin\omega_{gc}T_s\right]^2}}\right]$$

K_p then can be determined.

Implementation of self-tuning PI control to the two tanks system:

Figure (2-2) shows the simulink model used to implement the self-tuning PI controller to the two tanks system at operating point of 3 and 6.

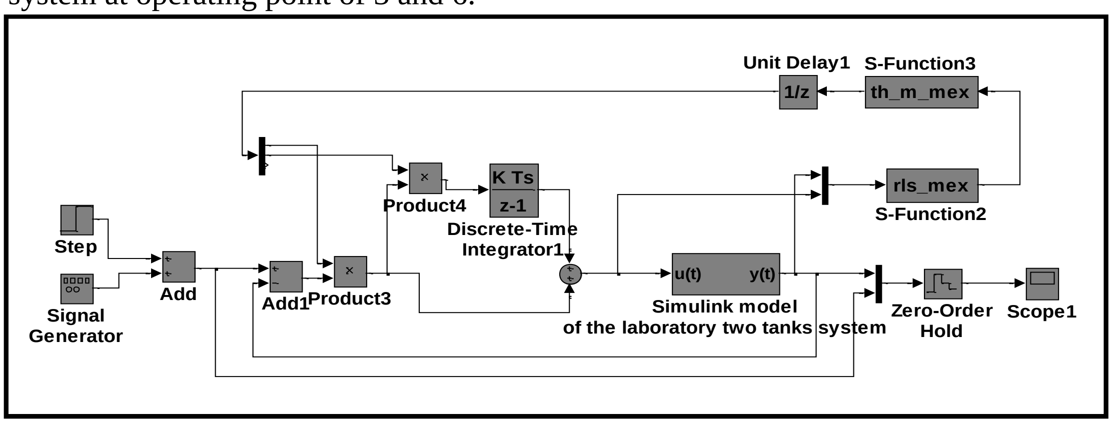

Figure (2-2) Self-tuning PI controller to the two tanks system.

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If we look at the figure in a bit of detail, it can be noted that the two tanks system is first being modelled by making use of the s-function2 block which represents the program file rls_mex.m. This program is the implementation of the RLS method. The model structure [na, nb, nk], the forgetting factor and the sampling time can be selected from the pop up window that appear in the s-function2. These parameters were chosen to be [1, 6, 1], [0.95], [40] as previously individually explained.

Controller parameters K_p *and* K_i calculation is the next step, where s-function3 is representing the th_m_mex.m program file which will calculate the K_p *and* K_i based on the steps explained in the theoretical section. The pop up window in the s-function3 shows the required parameters which are the phase margin (ϕ_m) in degrees, the sampling time in seconds, the initial controller settings K_p *and* K_i and the update interval in samples. These parameters were selected as follow [60], [40], [0.1 0.1], [400]. The next figure shows the output response at operating points of 3 and 6

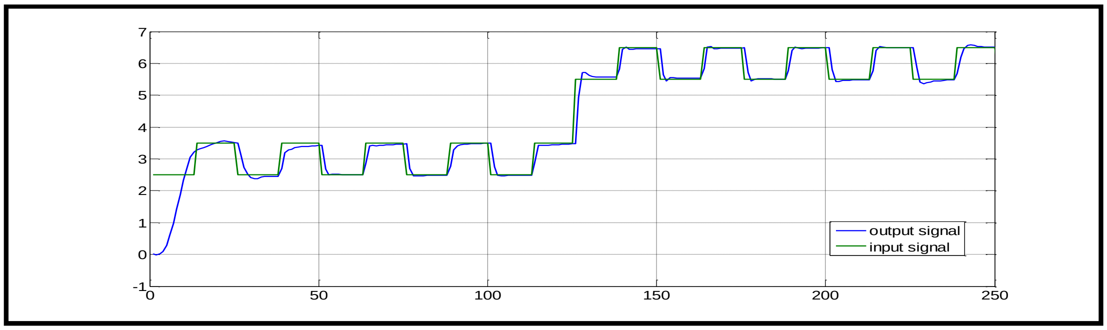
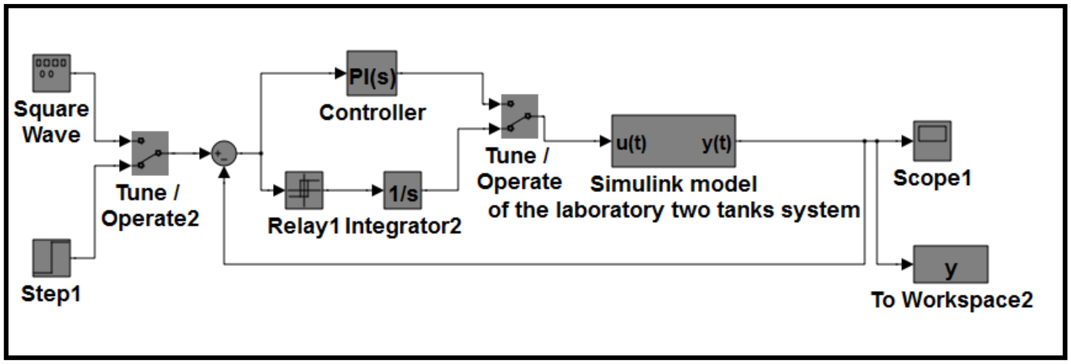


Figure (2-3) Output of the system with self-tuning PI controller.

Implementation of relay-based automatic PI controller to the two tanks system:
The next Simulink model figure (2-4) contains the scheme of applying the relay-based auto-tuning PI controller to the two tanks system. There are two selecting switches, the first is to alternate from a step input to a square wave input and the other is to alternate between the PI controller (with unknown (K_p *and* K_i) and the relay plus integrator.



Figure(2-4) Relay-based auto-tuning PI controller to the two tanks system.

The first step is connecting the step input and connecting the relay plus integrator to the two tanks system.
The next oscillation was obtained after several trials by changing the amplitude of the relay h.

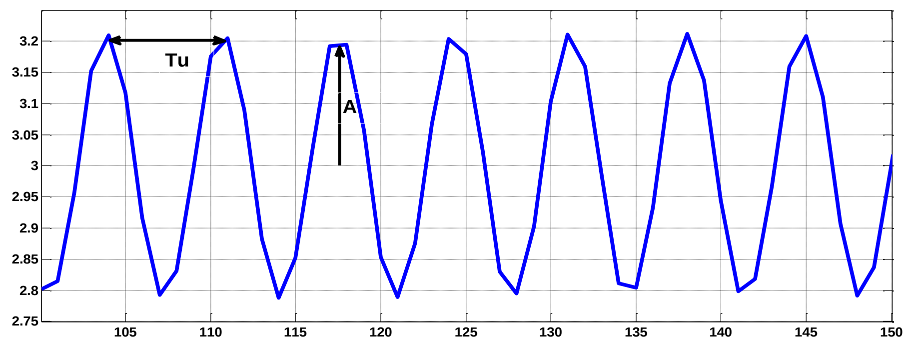


Figure (2-5) Oscillation of the system output using a relay plus integrator.

From the graph, the next results were measured based on relay amplitude $h = 0.01$:
$A = 0.212$ and $T_u = 275$
Then the approximate equivalent linear gain K_u for the relay can be as follow:

$$K_u = \frac{4h}{\pi A} = 0.06$$

And the ultimate frequency ω_u will be:

$$\omega_u = \frac{2\pi}{T_u} = 0.022$$

Finally the PI controller parameters can be calculated using the equations (2-13) and (2-14) where the phase margin ϕ_m is chosen to be 60^0 as below:

$$K_p = \frac{K_u}{\omega_u} sin\phi_m = 2.36$$

$$t_i = \frac{T_u}{2\pi} tan\phi_m = 75.8 \quad K_i = \frac{1}{t_i} = \frac{1}{75.8} = 0.013$$

The PI controller was set to these values and connected to the two tanks system and the square wave input through the selecting switches. The next graph shows the response of the two tanks system to this type of controlling where a good it shows a good tracking to the input signal.

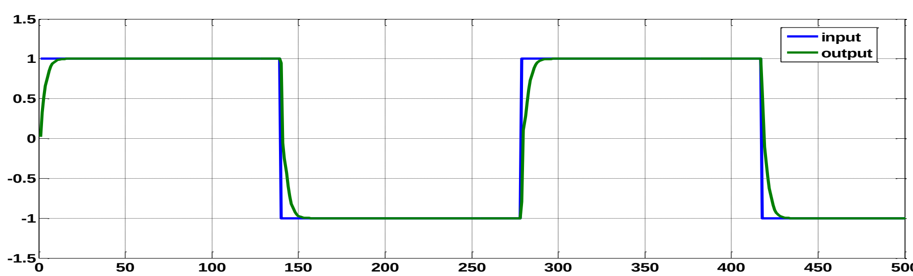


Figure (2-6) Output response to the relay-based auto-tuning PI controller.

The same steps were done to the system at an operating point of 6 and the results are as follow with the output response of the system:

$A = 0.427$ and $T_u = 390$

And consequently:

$K_u = \frac{4h}{\pi A} = 0.029$ and $\omega_u = \frac{2\pi}{T_u} = 0.016$

Hence:

$$K_p = \frac{K_u}{\omega_u} sin\phi_m = 1.57$$

$t_i = \frac{T_u}{2\pi} tan\phi_m = 107.56 \implies K_i = \frac{1}{t_i} = \frac{1}{75.8} = 0.0093$

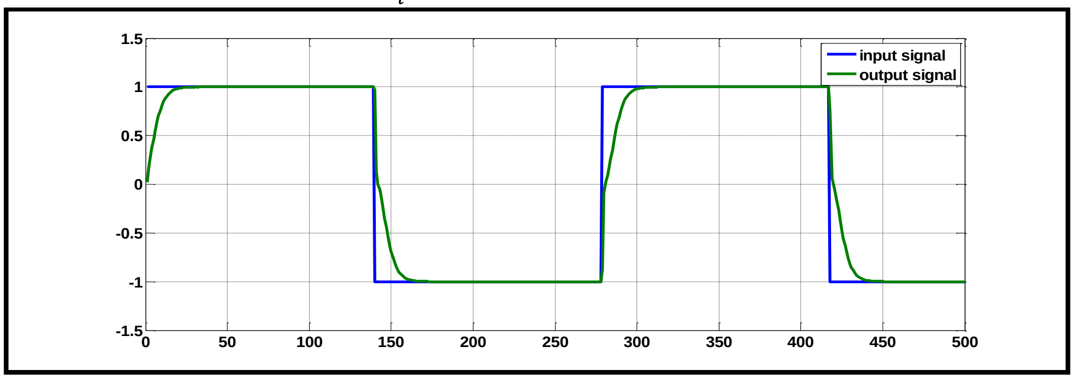


Figure (2-7) Output response to the relay-based auto-tuning PI controller.

Comparison between the two types of the PI controller:

The next Simulink model figure (2-8) shows the two types comparator model where the same input signal is applied to the two different PI controllers. Figures (2-9) and (2-10) shows the response of the system at operating points of 3 and 6 and vice versa.

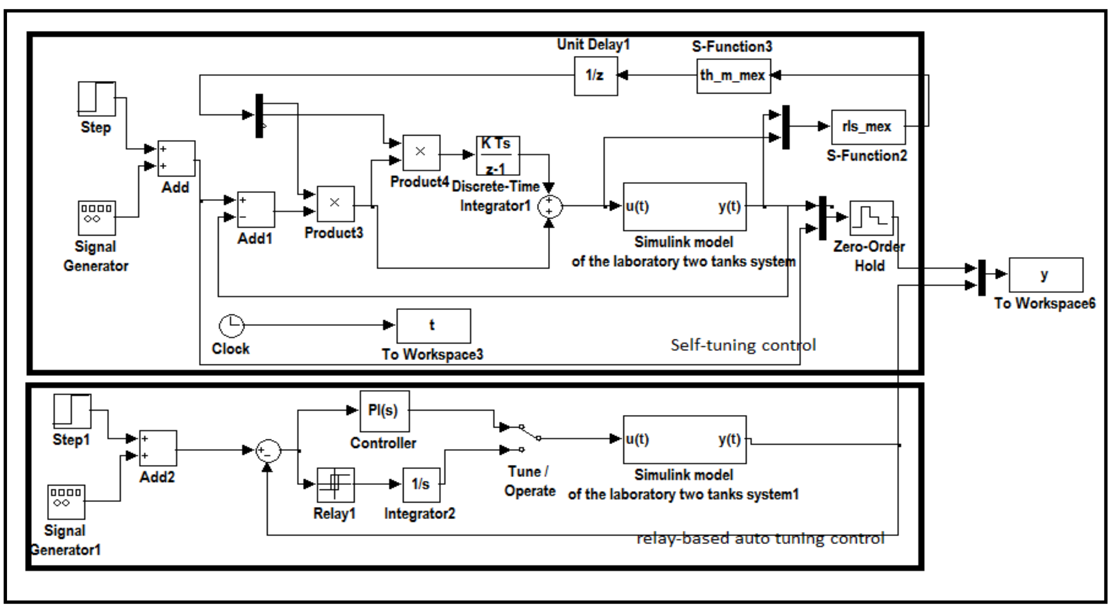


Figure (2-8) Simulink model for the two types of controlling comparator.

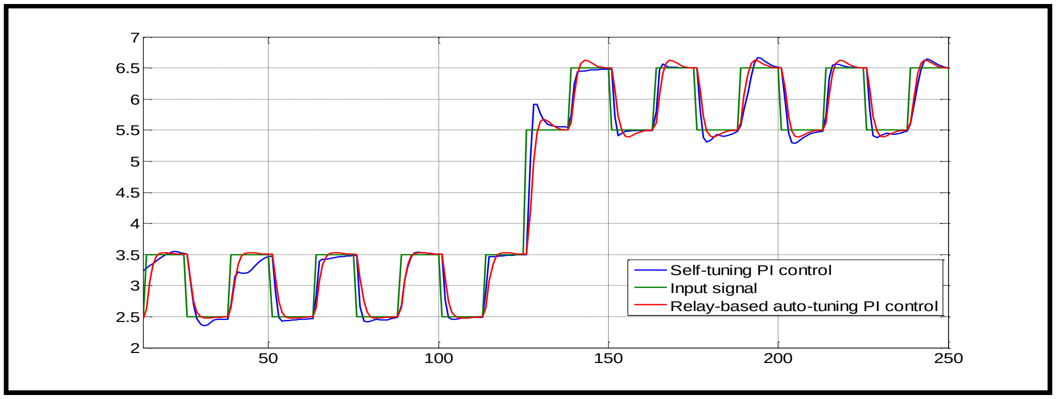


Figure (2-9) Comparison between the two types of controlling at operating points from 3 to 6.

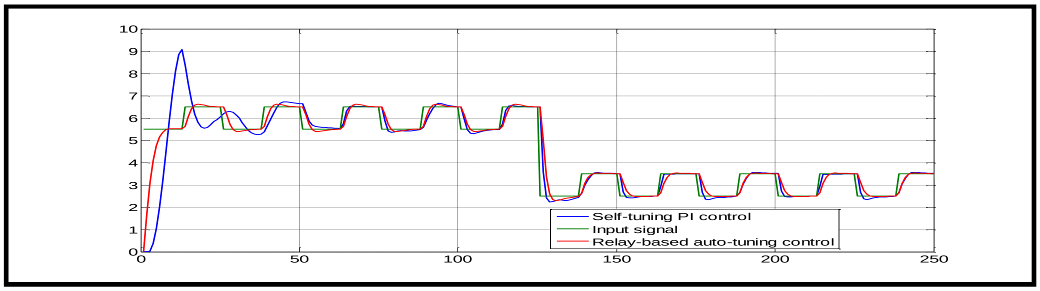


Figure (2-10) Comparison between the two types of controlling at operating points from 6 to3

Analysis of figure (2-9):

It can be noted from this figure that self-tuning PI control has fluctuating response at a frequency of 0.001Hz where the overshoot and speed of tracking the input signal vary and respond differently specially at the beginning of the two step input signals. The output reaches a steady response at the end of each operating point and that is because of the iterative computations using the recursive least squares method and, as it was explained, the forgetting factor helps to achieve a steady response quickly.

On the other hand, relay-based automatic tuning PI control shows a steady pace of tracking the input signal even when the input changes from 3 to 6 with more overshoot at operating point of 6 than it is at 3.

Analysis of figure (2-10):

The self-tuning PI control shows a very slow tracking to the input signal of 6 where a high overshoot can be witnessed at the beginning of applying the input signal until it begins to track the input signal after approximately two cycles of the input signal. At an operating point of 3, however, an acceptable response can be observed from the beginning of the input signal of 3.

In relation to the automatic tuning PI control, the response pace is still the same where a smooth tracking to the input signal with a small overshoot can be noted.

Conclusion

In this research, it has been given a clear picture of identifying the non-linear two tanks system using two different approaches, ordinary least squares and instrumental variable with the aid of YIC criterion of selecting the model structure where the system had been quantified and qualified before it was identified.

Matlab identification toolbox was utilized to identify the system where both discrete and continuous linear transfer function models were obtained. Outstanding results were achieved when the output of these models compared with the real two tanks system.

Self-tuning and relay-based auto- tuning PI controllers were designed successfully. Both of them were applied to the two tanks system and showed different impact to the system with acceptable responses. However, relay-based auto-tuning PI control showed a more stable response irrespective of the rapid input signal change.

Generally speaking, system identification and adaptive control were implemented successfully to the two tanks system with the theoretical support for better understanding.

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